CALCULUS AB
REVIEW FOR FIRST SEMESTER EXAM

Work these on **notebook paper**. Do not use your calculator.

Find the limit.

1. $\lim\limits_{x\to -3} \dfrac{x^2-2x-15}{x^2+4x+3} = \lim\limits_{x\to -3} \dfrac{(x-5)\cancel{(x+3)}}{(x+1)\cancel{(x+3)}} = \boxed{\dfrac{-3-5}{-3+1}} = \boxed{\dfrac{-8}{-2}} = \boxed{4}$

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2. $\lim\limits_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7} \cdot \dfrac{\sqrt{x+2}+3}{\sqrt{x+2}+3} = \lim\limits_{x\to 7} \dfrac{\overset{1}{\cancel{(x+2)-9}}}{\cancel{(x-7)}(\sqrt{x+2}+3)} = \boxed{\dfrac{1}{3+3}} = \boxed{\dfrac{1}{6}}$

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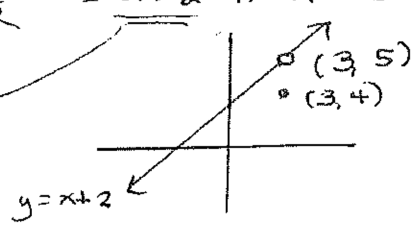
3. $f(x)=\begin{cases} \dfrac{x^2-x-6}{x-3} & \text{if } x\neq 3 \\ 4 & \text{if } x=3 \end{cases} = \dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = x+2 \text{ if } x\neq 3$



$\lim\limits_{x\to 3} f(x) = 3+2 = \boxed{5}$

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4. $\lim\limits_{x\to 0} \dfrac{\frac{1}{5+x}-\frac{1}{5}}{x} \cdot \dfrac{5(5+x)}{5(5+x)} = \lim\limits_{x\to 0} \dfrac{5-(5+x)}{5x(5+x)} = \lim\limits_{x\to 0} \dfrac{\overset{-1}{\cancel{-x}}}{5\cancel{x}(5+x)}$

$= \boxed{\dfrac{-1}{5(5+0)}} = \boxed{-\dfrac{1}{25}}$

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5. $\lim\limits_{x\to 0} \dfrac{\sin x}{3x^2+4x} = \lim\limits_{x\to 0} \dfrac{\sin x}{x(3x+4)} = \lim\limits_{x\to 0} \left(\dfrac{\sin x}{x} \cdot \dfrac{1}{3x+4}\right) = (1)\left(\dfrac{1}{4}\right)$

$\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$

$= \dfrac{1}{4}$

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6. $\lim\limits_{x\to 0} \dfrac{2x+\sin x}{5x} = \lim\limits_{x\to 0} \dfrac{2\cancel{x}}{5\cancel{x}} + \lim\limits_{x\to 0} \dfrac{\sin x}{5x}$

$= \lim\limits_{x\to 0} \dfrac{2}{5} + \dfrac{1}{5}\left(\lim\limits_{x\to 0} \dfrac{\sin x}{x}\right) = \dfrac{2}{5} + \dfrac{1}{5}(1) = \dfrac{3}{5}$

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7. $\lim\limits_{x\to -\infty} \dfrac{5x^3-4}{2x^3+1} = \lim\limits_{x\to -\infty} \dfrac{\frac{5x^3}{x^3}-\frac{4}{x^3}}{\frac{2x^3}{x^3}+\frac{1}{x^3}} = \lim\limits_{x\to -\infty} \dfrac{5-\frac{4}{x^3}}{2+\frac{1}{x^3}} = \dfrac{5-0}{2+0} = \boxed{\dfrac{5}{2}}$

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8. $\lim\limits_{x\to -\infty} \dfrac{2x^2-5}{3x+1} = \lim\limits_{x\to -\infty} \dfrac{\frac{2x^2}{x}-\frac{5}{x}}{\frac{3x}{x}+\frac{1}{x}} = \lim\limits_{x\to -\infty} \dfrac{2x-\frac{5}{x}}{3+\frac{1}{x}} = -\infty$

9. $\lim_{x\to -\infty} \frac{5x-7}{4x^2+3} =$ $\lim_{x\to -\infty} \frac{\frac{5x}{x^2} - \frac{7}{x^2}}{\frac{4x^2}{x^2} + \frac{3}{x^2}} = \lim_{x\to -\infty} \frac{\frac{5}{x} - \frac{7}{x^2}}{4 + \frac{3}{x^2}} = \frac{0-0}{4+0} = \boxed{0}$

Find the derivative. Do not leave negative exponents or complex fractions in your answers.

10. $f(x) = 3x^4 + \frac{2}{x} - 8x^{3/4} - 5$ $= 3x^4 + 2x^{-1} - 8x^{3/4} - 5$

$f'(x) = 12x^3 - 2x^{-2} - 6x^{-1/4} = \boxed{12x^3 - \frac{2}{x^2} - \frac{6}{x^{1/4}}}$

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11. $y = 3x^2 \cos(5x)$

$y' = (3x^2)(-5\sin(5x)) + (\cos(5x))(6x)$

$= -15x^2 \sin(5x) + 6x\cos(5x)$

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12. $f(x) = \frac{\tan x}{x^3}$

$f'(x) = \frac{x^3 \sec^2 x - 3x^2 \tan x}{x^6} = \boxed{\frac{x \sec^2 x - 3\tan x}{x^4}}$

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13. $y = (2x^3+5)^4$

$y' = 4(2x^3+5)^3(6x^2) = 24x^2(2x^3+5)^3$

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14. $f(x) = \sin(x^2)$

$f'(x) = \underbrace{(\cos(x^2))}_{\text{deriv of trig}} \underbrace{(2x)}_{\text{deriv of angle}} = \boxed{2x\cos(x^2)}$

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15. $y = \sin^3(5x)$ $= (\sin(5x))^3$

$y' = \overbrace{3(\sin(5x))^2}^{\text{Power}} \overbrace{(\cos(5x))}^{\text{Trig}} \overbrace{(5)}^{\text{Angle}}$

$= 15\sin^2(5x)\cos(5x)$

Evaluate the given integrals.

16. $\int(3x+1)(2x-5)\,dx = \int (6x^2 - 15x + 2x - 5)\,dx$

$= \int (6x^2 - 13x - 5)\,dx$

$= 2x^3 - \frac{13x^2}{2} - 5x + C$

17. $\int(5+\sec x\tan x - \sec^2 x)\,dx = 5x + \sec x - \tan x + C$

18. $\int \frac{x-4}{\sqrt{x^2-8x+1}}\,dx = \int (x^2-8x+1)^{-\frac{1}{2}}(x-4)\,dx = \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$u = x^2 - 8x + 1$

$du = (2x-8)\,dx$

$\frac{1}{2}\,du = (x-4)\,dx$

$= \frac{1}{\cancel{2}} \cdot \cancel{2} u^{\frac{1}{2}} + C$

$= (x^2-8x+1)^{1/2} + C$

or $\sqrt{x^2-8x+1} + C$

19. $\int x^2\cos(2x^3)\,dx = \frac{1}{6}\int \cos u\,du = \frac{1}{6}\sin u + C$

$u = 2x^3$

$du = 6x^2\,dx$

$\frac{1}{6}\,du = x^2\,dx$

$= \frac{1}{6}\sin(2x^3) + C$

20. $\int \sin^3(5x)\cos(5x)\,dx = \frac{1}{5}\int u^3\,du = \frac{1}{5}\cdot\frac{u^4}{4} + C$

$u = \sin(5x)$

$du = 5\cos(5x)\,dx$

$\frac{1}{5}\,du = \cos(5x)\,dx$

$= \frac{1}{20}\sin^4(5x) + C$

21. $\int_{\pi/12}^{\pi/9} \sin(3x)\,dx = \frac{1}{3}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin u\,du = -\frac{1}{3}\left[\cos u\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}\,du = dx$

$= -\frac{1}{3}\left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right)$

22. $\int_1^2 x(x^2+1)^3\,dx = \frac{1}{2}\int_2^5 u^3\,du = \frac{1}{2}\left[\frac{u^4}{4}\right]_2^5$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$= \frac{1}{8}\left[u^4\right]_2^5$

$= \boxed{\frac{1}{8}(625-16)} = \frac{1}{8}(5^4 - 2^4)$

$= \boxed{\frac{609}{8}}$

Multiple Choice. Show all work.

23. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

(A) undefined
(B) continuous but not differentiable
(C) differentiable but not continuous
(D) neither continuous nor differentiable
(E) both continuous and differentiable

$\lim_{x \to 3^-} (x^2) = 9$

$\lim_{x \to 3^+} (6x - 9) = 9$

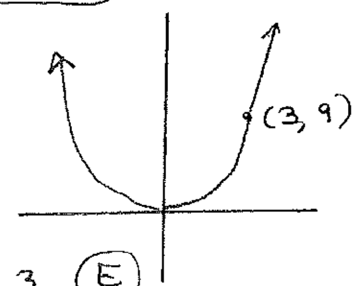
$\therefore$ f is continuous at $x = 3$.

$f'(x) = \begin{cases} 2x, & x < 3 \\ 6, & x > 3 \end{cases}$

$\lim_{x \to 3^-} (2x) = 6$

$\lim_{x \to 3^+} (6) = 6$

$\therefore$ f is differentiable at $x = 3$. E



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24. If $f(x) = \sin(2x)$, then $f'\left(\frac{\pi}{3}\right) =$

(A) -1 (B) $-\frac{1}{2}$ (C) $\sqrt{3}$ (D) 1 (E) $\frac{\sqrt{3}}{2}$

$f'(x) = 2\cos(2x)$

$f'\left(\frac{\pi}{3}\right) = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$ A

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25. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$ (D) $y = 3x - 1$ (E) $y = 4x + 1$

$y' = 3x^2 + 6x$

$y'' = 6x + 6 = 6(x+1) = 0$ when $x = -1$

y''	$-$		$+$
y	$\cap$	-1	$\cup$

When $x = -1$, $\begin{cases} y = -1 + 3 + 2 = 4 & \text{so } (-1, 4) = \text{point} \\ y' = 3 - 6 = -3 = \text{slope} \end{cases}$

Tangent line: $y - 4 = -3(x+1)$

$y - 4 = -3x - 3 \longrightarrow \boxed{y = -3x + 1}$ B

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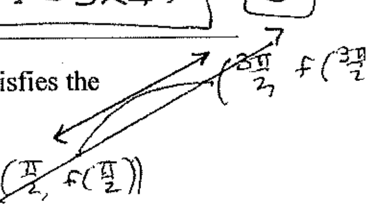
26. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?



(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

$\rightarrow f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

$\rightarrow \dfrac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \dfrac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{\pi} = 0$

$\frac{1}{2}\cos\left(\frac{x}{2}\right) = 0$

$\cos\left(\frac{x}{2}\right) = 0$

$\frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$

$x = \pi, 3\pi$. D

$c = \pi$

27. $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$ $\int_0^{\frac{\pi}{2}} (1+\sin\theta)^{-\frac{1}{2}} \cos\theta\, d\theta =$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

$u = 1+\sin\theta$

$du = \cos\theta\, d\theta$

$\int_1^2 u^{-\frac{1}{2}}\, du = 2\left[u^{\frac{1}{2}}\right]_1^2 = 2\left(\sqrt{2}-1\right)$ D

When $x = 0$, $u = 1+\sin 0 = 1+0 = \underline{\underline{1}}$

When $x = \frac{\pi}{2}$ $u = 1+\sin\frac{\pi}{2} = 1+1 = \underline{\underline{2}}$

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28. $\lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4}+h\right) - \tan\frac{\pi}{4}}{h} =$ $f'\left(\frac{\pi}{4}\right)$ where $f(x) = \tan x$

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) 1 (D) $\sqrt{2}$ (E) 2

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = \sec^2\frac{\pi}{4} = \left(\sqrt{2}\right)^2 = 2$

E

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

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29. Find $\frac{dy}{dx}$ given $x^2 + 3xy + y^3 = 10$

(A) $-\frac{2x+3y}{3x+3y^2}$ (B) $\frac{2x-3y}{3x+3y^2}$ (C) $-\frac{x+y}{x+y^2}$ (D) $\frac{x-y}{x+y^2}$

$2x + \underbrace{3x\frac{dy}{dx} + 3y} + 3y^2\frac{dy}{dx} = 0$

$(3x+3y^2)\frac{dy}{dx} = -2x-3y$

$\frac{dy}{dx} = \frac{-2x-3y}{3x+3y^2} = -\frac{2x+3y}{3x+3y^2}$ A

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30. Find the values of x that give relative extrema for $f(x) = \frac{1}{4}x^4 - 8x^2$.

(A) relative maximum: $x = -4$; relative minimum: $x = 0$ and $x = 4$

(B) relative maximum: $x = 0$; relative minimum: $x = -4$ and $x = 4$

(C) relative maximum: $x = -4$ and $x = 4$; relative minimum: $x = 0$

(D) relative maximum: $x = 4$; relative minimum: $x = 0$ and $x = -4$

$f'(x) = x^3 - 16x = x(x^2-16) = x(x+4)(x-4)$

$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	$\searrow$	-4	$\nearrow$	0	$\searrow$	4	$\nearrow$

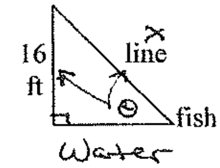
rel. max. at $x = 0$

rel. min. at $x = -4$ and $x = 4$

B

Free Response.

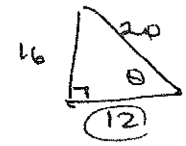
31. A fish is reeled in at a rate of 2 ft/sec from a bridge that is 16 ft. above the water. At what rate is the angle between the line and the water changing when there are 20 ft of line out?



Given: $\frac{dx}{dt} = -2 \frac{ft}{sec}$

Find: $\frac{d\theta}{dt}$ when $x = 20$ ft.

$\boxed{\sin\theta = \frac{16}{x} = 16x^{-1}}$



$3, 4, 5$
$\times\ 4$
$12, 16, 20$

$\cos\theta = \frac{12}{20} = \frac{3}{5}$

$\cos\theta \frac{d\theta}{dt} = -16x^{-2}\frac{dx}{dt} = -\frac{16}{x^2}\frac{dx}{dt}$

$\frac{3}{5}\frac{d\theta}{dt} = -\frac{16}{20^2}(-2)$

$\frac{3}{5}\frac{d\theta}{dt} = \frac{32}{400}$ Mult. by $\frac{5}{3}$

$\frac{d\theta}{dt} = \frac{5}{3} \cdot \frac{32}{400} = \boxed{\frac{2}{15} \frac{rad}{sec}}$

$\boxed{\left(\frac{5}{3} \cdot -\frac{16}{20^2} \cdot -2\right) \frac{rad}{sec}}$

32. A snowball is in the shape of a sphere. Its volume is increasing at a constant rate of $10\ \text{in}^3/\text{min}$. How fast is the radius increasing when the volume is $36\pi\ \text{in}^3$?

(Volume of a sphere: $V = \frac{4}{3}\pi r^3$)

$V = \frac{4}{3}\pi r^3$

Given: $\frac{dV}{dt} = +10 \frac{in^3}{min}$

Find: $\frac{dr}{dt}$ when $V = 36\pi\ in^3$

$\boxed{36\pi = \frac{4}{3}\pi r^3 \quad 27 = r^3 \quad 3 = r}$

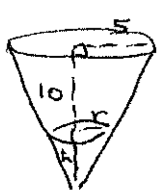
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$10 = 4\pi(9)\frac{dr}{dt}$

$10 = 36\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \boxed{\frac{10}{36\pi} \frac{in}{min}} = \boxed{\frac{5}{18\pi} \frac{in}{min}}$

33. Water runs out of a conical tank at the constant rate of 2 cubic feet per minute. The radius at the top of the tank is 5 feet, and the height of the tank is 10 feet. How fast is the water level sinking when the water is 4 feet deep? (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$)



Given: $\frac{dV}{dt} = -2 \frac{ft^3}{min}$

Find: $\frac{dh}{dt}$ when $h = 4$ ft

$\frac{r}{h} = \frac{5}{10} = \frac{1}{2}$

$2r = h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)(h)$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$-2 = \frac{1}{4}\pi(16)\frac{dh}{dt}$

$-2 = 4\pi \frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{2}{4\pi} = \boxed{-\frac{1}{2\pi} \frac{ft}{min}}$

The water is sinking at a rate of $\frac{1}{2\pi} \frac{ft}{min}$.

34. Consider the curve defined by $y^2 + \underline{xy} - x^3 = 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y..

$$2y\frac{dy}{dx} + \underline{x\frac{dy}{dx} + y} - 3x^2 = 0$$

$$(2y + x)\frac{dy}{dx} = 3x^2 - y$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2 - y}{2y + x}}$$

(b) Evaluate $\frac{dy}{dx}$ at the point $(2, -4)$

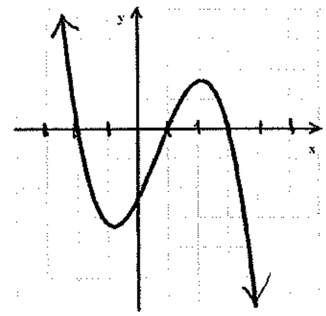
At $(2, -4)$, $\frac{dy}{dx} = \frac{12 - (-4)}{-8 + 2} = \frac{16}{-6} = \boxed{-\frac{8}{3}}$

(c) Write the equation of the tangent line to the curve $y^2 + xy - x^3 = 8$ at the point $(2, -4)$.

$$\boxed{y + 4 = -\frac{8}{3}(x - 2)}$$

35. ~~36~~. The graph of $y = f'(x)$ is shown on the right.

(a) On what interval(s) is the graph of f decreasing? Justify your answer.



Graph of $y = f'(x)$

$f'(x)$ + | − | + | −

$f(x)$ ↗ −2 ↘ 1 ↗ 3 ↘

$\boxed{\text{f is decr. on } (-2, 1) \text{ and } (3, \infty) \text{ b/c } f'(x) < 0 \text{ there.}}$

(b) For what value(s) of x does the graph of f have a local maximum? Justify your answer in sentence.

$\boxed{\text{f has a local max. at } x = -2 \text{ and at } x = 3 \text{ b/c } f'(x) \text{ changes from positive to negative there.}}$

(c) On what interval(s) is the graph of f concave upward? Justify your answer.

$f'(x)$ ↘ | ↗ | ↘

$f(x)$ ∩ −1 ∪ 2 ∩

$\boxed{\text{f is concave up on } (-1, 2) \text{ b/c } f'(x) \text{ is increasing there.}}$

(d) For what value(s) of x does the graph of f have an inflection point? Justify your answer in sentence.

$\boxed{\text{f has an IP at } x = -1 \text{ and } x = 2 \text{ b/c } f'(x) \text{ changes from decr. to incr. or vice versa there.}}$

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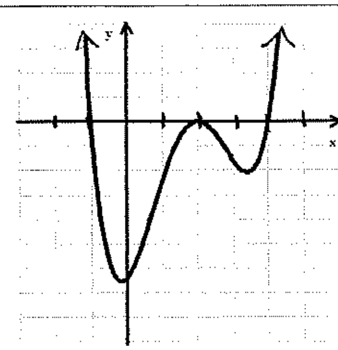
36. ~~37~~. The graph of $y = f''(x)$ is shown on the right.

(a) On what interval(s) is the graph of f concave downward? Justify your answer.



Graph of $y = f''(x)$

$f''(x)$ + | − | − | +

$f(x)$ ∪ −1 ∩ 2 ∩ 4 ∪

$\boxed{\text{f is C.D. on } (-1, 2) \text{ and } (2, 4) \text{ b/c } f''(x) < 0 \text{ there.}}$

(b) At what value(s) of x does the graph of f have an inflection point? Justify your answer in a sentence.

$\boxed{\text{f has an IP at } x = -1 \text{ and } x = 4 \text{ b/c } f''(x) \text{ changes from positive to negative or vice versa there.}}$

37. ~~38~~. Let $y(t)$ represent the temperature of a pie that has been removed from a 450° F oven and left to cool in a room with a temperature of 72° F, where y is a differentiable function of t. The table below shows the temperature recorded every five minutes.

t (min.)	0	5	10	15	20	25	30
$y(t)$ (° F)	450	388	338	292	257	226	200

(a) Use data from the table to find an approximation for $y'(18)$, and explain the meaning of $y'(18)$ in terms of the temperature of the pie. Show the computations that lead to your answer, and indicate units of measure.

$$y'(18) \approx \frac{257-292}{20-15} = -\frac{35}{5} = \boxed{-7^\circ \text{ F/min}}$$

When $t = 18$ min, the temperature of the pie is decreasing at a rate of 7° F per minute.

(b) For $0 < t < 30$, must there be a time t when the temperature of the pie is 260°F? Justify your answer.

Yes. Since $y(t)$ is continuous, $y(15) = 292$ and $y(20) = 257$ and 260 lies between 257 and 292, there must be at least one t in $(15, 20)$ where $y(t) = 260°$ F (by the Intermediate Value Theorem.)

There is not a prob. 38.

38. ~~39~~. The rate at which water is being pumped into a tank is given by the increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 \le t \le 20$ minutes, is shown below.

t (min.)	0	4	9	17	20
$R(t)$ (gal/min)	25	28	33	42	46

(a) Use a left Riemann sum with four subintervals to approximate the value of $\int_0^{20} R(t)\,dt$.

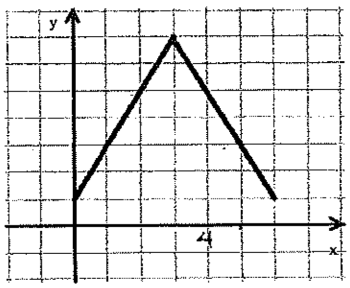
$$\int_0^{20} R(t)\,dt \approx 4(25) + 5(28) + 8(33) + 3(42) \text{ gal}$$
$$= 630 \text{ gal}$$

(b) Use a right Riemann sum with four subintervals to approximate the value of $\int_0^{20} R(t)\,dt$.

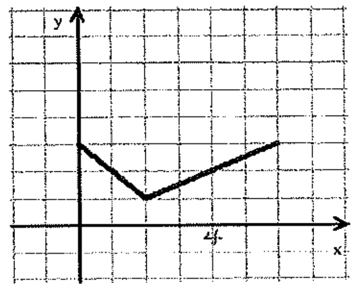
$$\int_0^{20} R(t)\,dt \approx 4(28) + 5(33) + 8(42) + 3(46) \text{ gal}$$
$$= 751 \text{ gal}$$

(c) Use a trapezoidal sum with four subintervals to approximate the value of $\int_0^{20} R(t)\,dt$.

$$\int_0^{20} R(t)\,dt \approx \tfrac{1}{2}(4)(25+28) + \tfrac{1}{2}(5)(28+33) + \tfrac{1}{2}(8)(33+42) + \tfrac{1}{2}(3)(42+46) \text{ gal}$$
$$= 690.5 \text{ gal}$$



Graph of f



Graph of g

Use the graphs above for problems 40 – 42.

40. If $h(x)=f(x)g(x)$, find $h'(4)$.

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(4) = \boxed{(5)(\tfrac{1}{2}) + (2)(-2)}$

$= \boxed{\tfrac{5}{2} - 4} = \boxed{-\tfrac{3}{2}}$

42. If $k(x)=g(f(x))$, find $k'(5)$.

$k'(x) = g'(f(x))\,f'(x)$

$k'(5) = g'(\underline{f(5)})\,f'(5)$

$= g'(\underline{3})\,f'(5)$

$= \boxed{(\tfrac{1}{2})(-2)} = \boxed{-1}$

41. If $m(x)=\dfrac{f(x)}{g(x)}$, find $m'(1)$.

$m'(x) = \dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}$

$m'(1) = \boxed{\dfrac{(2)(2) - (3)(-1)}{2^2}}$

$= \boxed{\dfrac{4+3}{4}} = \boxed{\dfrac{7}{4}}$

---

43. Find $\dfrac{dy}{dx}$ in terms of x and y, given $\cos x + 3\sin(2y) = 5$.

$\cos x + 3\sin(2y) = 5$

$-\sin x + 6\cos(2y)\dfrac{dy}{dx} = 0$

$6\cos(2y)\dfrac{dy}{dx} = \sin x$

$\boxed{\dfrac{dy}{dx} = \dfrac{\sin x}{6\cos(2y)}}$

44. Find $\frac{d^2y}{dx^2}$ in terms of x and y, given $\frac{dy}{dx}=\frac{5x}{3x+4y}$.

$$\frac{d^2y}{dx^2}=\frac{(3x+4y)(5)-(5x)\left(3+4\frac{dy}{dx}\right)}{(3x+4y)^2}$$

$$=\frac{\cancel{15x}+20y-\cancel{15x}-20x\left(\frac{5x}{3x+4y}\right)}{(3x+4y)^2}\cdot\frac{3x+4y}{3x+4y}$$

$$=\boxed{\frac{60xy+80y^2-100x^2}{(3x+4y)^3}}$$

---

45. Show whether or not the conditions of the Mean Value Theorem are met. If the theorem applies, find the value of c that the Mean Value Theorem guarantees.

$$f(x)=\frac{1}{x-1}\quad [2,5]$$

f is continuous on $[2,5]$.

f is differentiable on $(2,5)$.

$\therefore$ MVT applies.

$f'(x)=-(x-1)^{-2}=\boxed{-\frac{1}{(x-1)^2}}$

$\frac{f(5)-f(2)}{5-2}=\frac{\frac{1}{4}-1}{3}=\frac{-\frac{3}{4}}{3}=-\frac{3}{4}\cdot\frac{1}{3}=\boxed{-\frac{1}{4}}$

$-\frac{1}{(x-1)^2}=-\frac{1}{4}$

$(x-1)^2=4$

$x-1=\pm 2$

$x=1\pm 2$

$x=3$ & -1 $\boxed{c=3}$

---

46. Given $f(x)=\frac{1}{3}x^3-x^2-3x+2$, find the intervals where f is increasing and decreasing, and identify all points that are relative maximum and minimum points. Justify your answers.

$f'(x)=x^2-2x-3=(x-3)(x+1)=0$ when $x=3$ & -1

$f'(x)$ + | − | +

$f(x)$ ↗ -1 ↘ 3 ↗

f is incr. on $(-\infty,-1)$ and $(3,\infty)$ b/c $f'(x)>0$ there.
f is decr. on $(-1,3)$ b/c $f'(x)<0$ there.

f has a rel. max. at $(-1,3\frac{2}{3})$ b/c $f'(x)$ changes from pos. to neg. there. f has a rel. min. at $(3,-7)$ b/c $f'(x)$ changes from neg. to pos. there.

47. Given $f(x)=\frac{1}{2}x^4+2x^3$, find the intervals where f is concave up and concave down, and find the inflection points. Justify your answer.

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x+2) = 0$ when $x = 0, -2$

$f''(x)$	+		−		+
$f(x)$	∪	−2	∩	0	∪

f is CU on $(-\infty, -2)$ and $(0, \infty)$ b/c $f''(x) > 0$ there,
f is CD on $(-2, 0)$ b/c $f''(x) < 0$ there,

f has an IP at $(-2, -8)$ and $(0, 0)$ b/c $f''(x)$ changes from pos. to neg. or vice versa.

48. Suppose that f'' is continuous on $(-\infty, \infty)$. If $f'(-3)=0$ and $f''(-3)=5$, what can you conclude about f by the Second Derivative Test?

$f'(-3) = 0$ so f has a horiz. tangent at $x = -3$.

$f''(-3) = 5$ so f is concave up at $x = -3$.

$\therefore$ f has a local minimum at $x = -3$ by the Second Derivative Test,

49. Given $f(x)=\frac{2}{3}x^3+\frac{1}{2}x^2-15x+9$. Use the Second Derivative Test to find whether f has a local maximum or a local minimum at $x=-3$. Justify your answer.

$f'(x) = 2x^2 + x - 15$

$f''(x) = 4x + 1$

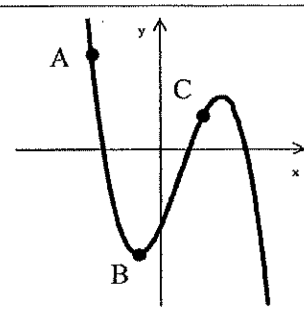
$f'(-3) = 18 - 3 - 15 = 0$ so f has a horiz. tangent at $x = -3$.

$f''(-3) = -12 + 1 = -11$ so f is concave down at $x = -3$.

$\therefore$ f has a local maximum at $x = -3$ by the Sec. Deriv. Test,

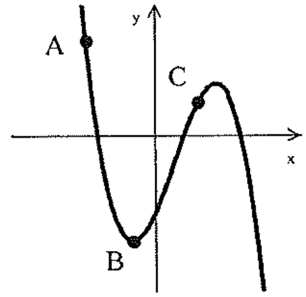
50. The graph of a function f is shown on the right.
Fill in the chart with +, −, or 0.

Point	f	f'	f''
A	+	−	+
B	−	0	+
C	+	+	−

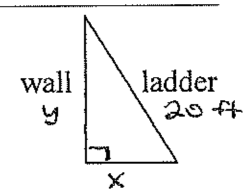


50. The graph of a function f is shown on the right. Fill in the chart with +, −, or 0.

Point	f	f'	f''
A	+	−	+
B	−	0	+
C	+	+	−

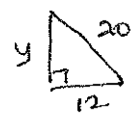


51. A ladder 20 ft long is leaning against the wall of a house. The base of the ladder is being pulled away from the wall at a rate of $2\frac{\text{ft}}{\text{sec}}$. How fast is the top of the ladder moving down the wall when the base of the ladder is 12 ft from the wall?



Given: $\frac{dx}{dt} = 2 \frac{ft}{sec}$

Find: $\frac{dy}{dt}$ when $x = 12$ ft.



$12^2 + y^2 = 20^2$

$y^2 = 256$

$y = 16$ ft.

3, 4, 5 × 4 = 12, 16, 20

$x^2 + y^2 = 20^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$ Divide by 2

$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$

$12(2) + 16 \frac{dy}{dt} = 0$

$16 \frac{dy}{dt} = -24$

$\boxed{\frac{dy}{dt} = -\frac{24}{16} = -\frac{3}{2} \frac{ft}{sec}}$

Evaluate.

55. $\int_0^1 x(x^2+1)^3\,dx = \frac{1}{2}\int_1^2 u^3\,du = \frac{1}{2}\left[\frac{u^4}{4}\right]_1^2 = \frac{1}{8}\left[u^4\right]_1^2$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \boxed{\frac{1}{8}(2^4 - 1^4)}$

$= \boxed{\frac{15}{8}}$

---

56. $\int \frac{2x}{\sqrt{x^2+4}}\,dx = \int 2x\,(x^2+4)^{-\frac{1}{2}}\,dx = \int u^{-\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$u = x^2+4$

$du = 2x\,dx$

$= 2u^{\frac{1}{2}} + C$

$= \boxed{2(x^2+4)^{\frac{1}{2}} + C}$

or $\boxed{2\sqrt{x^2+4} + C}$

---

57. $\int_{-\pi/12}^{\pi/6} \sin(2x)\,dx = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \sin u\,du = -\frac{1}{2}\left[\cos u\right]_{-\frac{\pi}{6}}^{\frac{\pi}{3}}$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$= -\frac{1}{2}\left(\cos\frac{\pi}{3} - \cos\left(-\frac{\pi}{6}\right)\right)$

$= \boxed{-\frac{1}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)}$

---

58. $\int_{\pi/6}^{\pi/2} \sin^3 x \cos x\,dx = \int_{\frac{1}{2}}^{1} u^3\,du = \left[\frac{u^4}{4}\right]_{\frac{1}{2}}^{1} = \frac{1}{4}\left[u^4\right]_{\frac{1}{2}}^{1}$

$u = \sin x$

$du = \cos x\,dx$

$= \boxed{\frac{1}{4}\left(1 - \frac{1}{16}\right)}$

$= \boxed{\frac{1}{4}\left(\frac{15}{16}\right)}$

$= \boxed{\frac{15}{64}}$

---

59. $\int x\sqrt{x+2}\,dx = \int (u-2)(u^{\frac{1}{2}})\,du = \int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right)du$

$u = x+2$

$x = u-2$

$dx = du$

$= \frac{u^{5/2}}{5/2} - \frac{2u^{3/2}}{3/2} + C$

$= \frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$

60. $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx = \int_1^9 \frac{\frac{u+1}{2}}{u^{\frac{1}{2}}} \cdot \frac{1}{2}\,du = \frac{1}{4}\int_1^9 \frac{u+1}{u^{\frac{1}{2}}}\,du \qquad \frac{u}{u^{\frac{1}{2}}} + \frac{1}{u^{\frac{1}{2}}}$

$u = 2x-1$

$x = \frac{u+1}{2} = \frac{1}{2}u + \frac{1}{2}$

$dx = \frac{1}{2}\,du$

$= \frac{1}{4}\int_1^9 (u^{\frac{1}{2}} + u^{-\frac{1}{2}})\,du$

$= \frac{1}{4}\left[\frac{u^{\frac{3}{2}}}{3/2} + \frac{u^{\frac{1}{2}}}{1/2}\right]_1^9$

$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_1^9$

$= \boxed{\frac{1}{4}\left(\left(\frac{2}{3}(9)^{3/2} + 2(9)^{1/2}\right) - \left(\frac{2}{3}+2\right)\right)} = \boxed{\frac{16}{3}}$

---

61. Let $f(x) = \begin{cases} 2x-5 & \text{for } x \le 3 \\ \sqrt{x+1} & \text{for } x > 3. \end{cases}$ Find $\int_0^8 f(x)\,dx$.

$\int_0^3 + \int_3^8$

$\int_0^8 f(x)\,dx = \int_0^3 (2x-5)\,dx + \int_3^8 \sqrt{x+1}\,dx \leftarrow \begin{matrix} u = x+1 \\ du = dx \end{matrix}$

$= \int_0^3 (2x-5)\,dx + \int_4^9 u^{\frac{1}{2}}\,du$

$= \left[x^2 - 5x\right]_0^3 + \frac{2}{3}\left[u^{3/2}\right]_4^9$

$= \boxed{(9-15) - (0-0) + \frac{2}{3}(9^{3/2} - 4^{3/2})} = \boxed{-6 + \frac{2}{3}(27-8)} = \boxed{\frac{56}{3}}$

On problems 62 and 63,

(a) Find the average value of f on the given interval.

(b) Find the value of c such that $f_{AVE} = f(c)$.

62. $f(x) = (x-3)^2$, $[2, 5]$

(a) $f_{AVE} = \frac{1}{3}\int_2^5 (x-3)^2\,dx = \frac{1}{3}\int_{-1}^2 u^2\,du = \frac{1}{3}\left[\frac{u^3}{3}\right]_{-1}^2 = \frac{1}{3}\left(\frac{8}{3} - -\frac{1}{3}\right) = [$

$u = x-3$

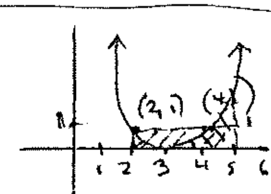
$du = dx$

(b) $(x-3)^2 = 1$

$x - 3 = \pm 1$

$x = 3 \pm 1$

$\boxed{x = 4 \;\&\; 2}$ $\boxed{c = 4 \;\&\; 2}$



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63. $f(x) = \sqrt{x}$, $[0, 4]$

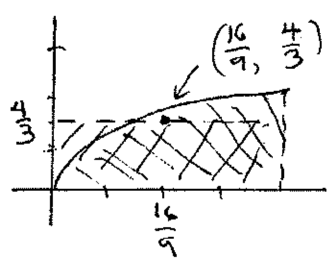
(a) $f_{AVE} = \frac{1}{4}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{1}{4}\cdot\frac{2}{3}\left[x^{3/2}\right]_0^4$

$= \boxed{\frac{1}{6}(4^{3/2} - 0)} = \boxed{\frac{1}{6}(8)} = \boxed{\frac{4}{3}}$

(b) $\sqrt{x} = \frac{4}{3}$

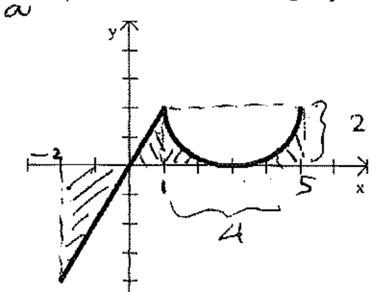
$x = \frac{16}{9}$

$\boxed{c = \frac{16}{9}}$



$\int_a^b f'(x)dx = f(b) - f(a)$

64. The graph of f' which consists of a line segment and a semicircle, is shown on the right. Given that $\boxed{f(1) = 4,}$ find:



Graph of f'

← subtr.

→ add.

(a) $f(-2)$ $\quad \int_{-2}^{1} f'(x)dx = f(1) - f(-2)$

$f(-2) = f(1) - \int_{-2}^{1} f'(x)dx =$

$= \boxed{4 - \left(\tfrac{1}{2}(1)(2) - \tfrac{1}{2}(4)(2)\right)} = \boxed{7}$

(b) $f(5)$ $\quad \int_1^5 f'(x)dx = f(5) - f(1)$

$f(5) = f(1) + \int_1^5 f'(x)dx$

$= \boxed{4 + ((4)(2) - 2\pi)}$

$= \boxed{4 + 8 - 2\pi} = \boxed{12 - 2\pi}$

---

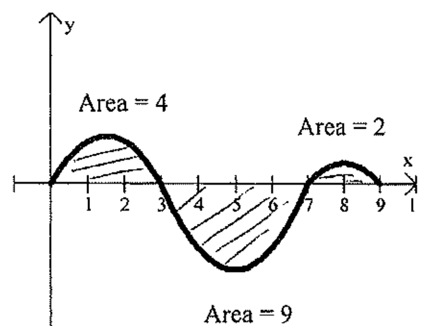
65. The graph of f' is shown. Use the figure and the fact that $\boxed{f(3) = 5}$ to find:

(a) $f(0)$

(b) $f(7)$

(c) $f(9)$

(d) Use the given information and your answers to (a), (b), and (c) to sketch the graph of f.



Graph of f'

$f'(x)$ + − +

$f(x)$ 0 ↗ 3 ↘ 7 ↗ 9

$f'(x)$ ↗ ↘ ↗ ↘

$f(x)$ 0 ∪ $1\tfrac{1}{2}$ ∩ 5 ∪ 8 ∩ 9

(a) $\int_0^3 f'(x)dx = f(3) - f(0)$

$f(0) = f(3) - \int_0^3 f'(x)dx$

$= 5 - {+4} = \boxed{1}$

(b) $\int_3^7 f'(x)dx = f(7) - f(3)$

$f(7) = f(3) + \int_3^7 f'(x)dx$

$= 5 + -9 = \boxed{-4}$

(c) $\int_7^9 f'(x)dx = f(9) - f(7)$

$f(9) = f(7) + \int_7^9 f'(x)dx$

$= -4 + 2 = \boxed{-2}$

x	f(x)
0	1
3	5
7	-4
9	-2