Answers to Summer Packet
(with Work Shown)

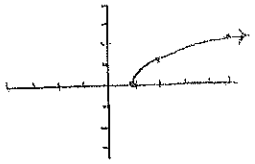
① $y = \sqrt{x-1}$

x-int.: (1, 0)

y-int.: none

$D = \{x : x \geq 1\}$

$R = \{y : y \geq 0\}$



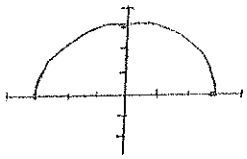
② $y = \sqrt{9-x^2}$

x-int.: (3, 0), (-3, 0)

y-int.: (0, 3)

$D = \{x : -3 \leq x \leq 3\}$

$R = \{y : 0 \leq y \leq 3\}$



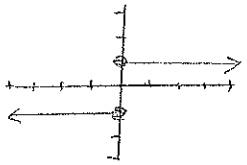
③ $y = \frac{|x|}{x}$

x-int.: none

y-int.: none

$D = \{x : x \neq 0\}$

$R = \{-1, 1\}$



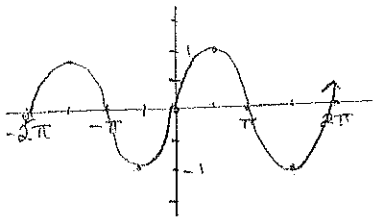
④ $y = \sin x, -2\pi \leq x \leq 2\pi$

x-int.: $(-2\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$

y-int.: (0, 0)

$D = \{x : -2\pi \leq x \leq 2\pi\}$

$R = \{y : -1 \leq y \leq 1\}$



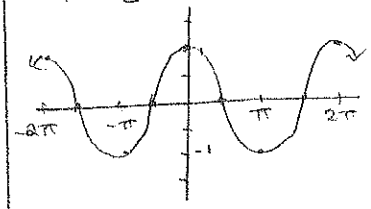
⑤ $y = \cos x$

x-int.: $(-\frac{3\pi}{2}, 0), (-\frac{\pi}{2}, 0), (\frac{\pi}{2}, 0), (\frac{3\pi}{2}, 0)$

y-int.: (0, 1)

$D = \{x : -2\pi \leq x \leq 2\pi\}$

$R = \{y : -1 \leq y \leq 1\}$



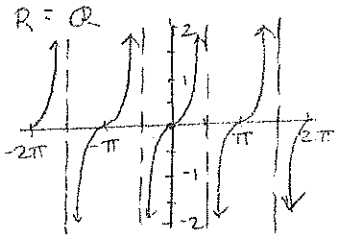
⑥ $y = \tan x$

x-int.: $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$

y-int.: (0, 0)

$D = \{x : -2\pi \leq x \leq 2\pi, x \neq -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\}$

$R = \mathbb{R}$



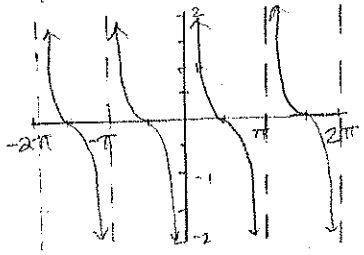
⑦ $y = \cot x$

x-int.: $(-\frac{3\pi}{2}, 0), (-\frac{\pi}{2}, 0), (\frac{\pi}{2}, 0), (\frac{3\pi}{2}, 0)$

y-int.: none

$D = \{x : -2\pi < x < 2\pi, x \neq -\pi, 0, \pi\}$

$R = \mathbb{R}$



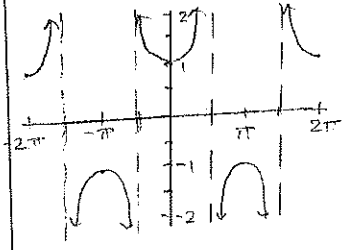
⑧ $y = \sec x$

x-int.: none

y-int.: (0, 1)

$D = \{x : -2\pi \leq x \leq 2\pi, x \neq -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\}$

$R = \{y : y \leq -1 \text{ or } y \geq 1\}$



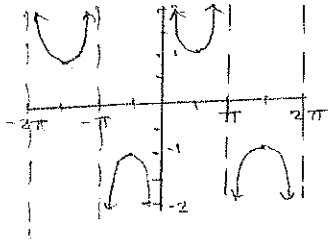
⑨ $y = \csc x$

x-int.: none

y-int.: none

$D = \{x : -2\pi < x < 2\pi, x \neq -\pi, 0, \pi\}$

$R = \{y : y \leq -1 \text{ or } y \geq 1\}$



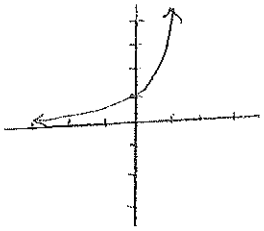
(10) $y = e^x$

x-int: none

y-int: $(0, 1)$

$D = \mathbb{R}$

$R = \{y : y > 0\}$



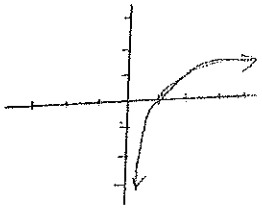
(11) $y = \ln x$

x-int: $(1, 0)$

y-int: none

$D = \{x : x > 0\}$

$R = \mathbb{R}$



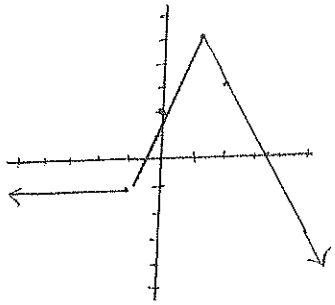
(12) $y = \begin{cases} -1, & \text{if } x \le -1 \\ 3x + 2, & \text{if } |x| < 1 \\ 7 - 2x, & \text{if } x \ge 1 \end{cases}$

x-int: $\left(\frac{7}{2}, 0\right), \left(-\frac{2}{3}, 0\right)$

y-int: $(0, 2)$

$D = \mathbb{R}$

$R = \{y : y \le 5\}$



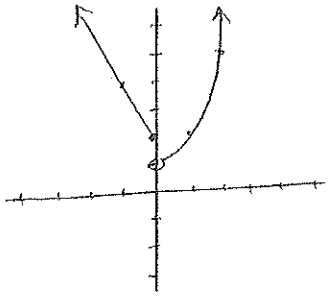
(13) $y = \begin{cases} x^2 + 1, & \text{if } x > 0 \\ -2x + 2, & \text{if } x \le 0 \end{cases}$

x-int: none

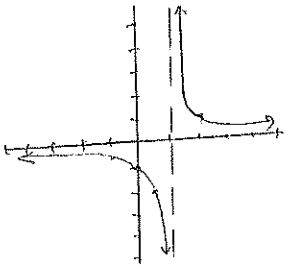
y-int: $(0, 2)$

$D = \mathbb{R}$

$R = \{y : y > 1\}$



(14) $y = \frac{1}{x - 1}$



Vert: $x = 1$

Horiz: $y = 0$

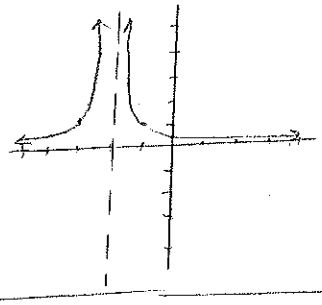
Slant: none

Sym.: none

x-int: none

y-int: $(0, -1)$

(15) $y = \frac{1}{(x + 2)^2}$



Vert: $x = -2$

Horiz: $y = 0$

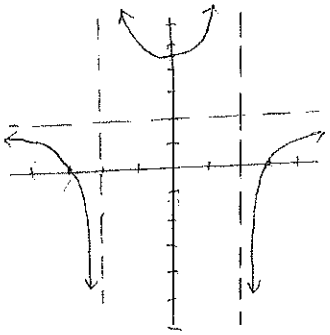
Slant: none

Sym.: none

x-int: none

y-int: $\left(0, \frac{1}{4}\right)$

(16) $y = \frac{2(x^2 - 9)}{x^2 - 4}$



Vert: $x = \pm 2$

Horiz: $y = 2$

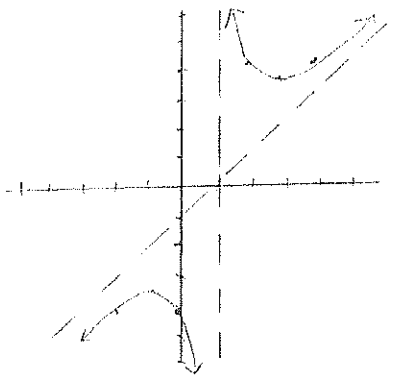
Slant: none

Sym.: y-axis

x-int: $(3, 0), (-3, 0)$

y-int: $\left(0, \frac{9}{2}\right)$

(17) $y = \frac{x^2 - 2x + 4}{x - 1}$



Vert: $x = 1$
Horiz: none
Slant: $y = x - 1$
Sym: none
x-int: none
y-int: $(0, -4)$

$$x-1 \;\big|\; x^2 - 2x + 4 \qquad x - 1$$
$$x^2 - x$$
$$-x + 4$$
$$-x + 1$$
$$3$$

x	y
2	4
3	7/2
4	4
0	-4
-1	-7/2
-2	-4

---

(18) $x^2 - x - 12 > 0$

$(x-4)(x+3) > 0$

(+) , (−) , (+)

-3 , 4

$x < -3$ or $x > 4$

(19) $(x-2)^2(x+1)^3(x-5) \le 0$

+ , ⊖ , ⊖ , +

-1 , 2 , 5

$-1 \le x \le 5$

(20) $\frac{3x-2}{x+4} \le 0$

+ , ⊖ , +

-4 , $\frac{2}{3}$

$-4 < x \le \frac{2}{3}$

(21) $\frac{(2x+5)(x-1)^2}{(x+2)^3} \ge 0$

(+) , − , (+) , (+)

$-\frac{5}{2}$, -2 , 1

$x \le -\frac{5}{2}$ or $x > -2$

---

(22) $\sec \frac{4\pi}{3} = -2$

(23) $\sin \frac{3\pi}{2} = -1$

(24) $\tan \frac{5\pi}{4} = 1$

(25) $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

(26) $\cos \pi = -1$

(27) $\tan \frac{2\pi}{3} = -\sqrt{3}$

(28) $\sec \frac{\pi}{3} = \frac{1}{\frac{1}{2}} = 2$

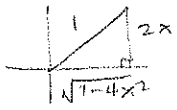
(29) $\csc \frac{\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(30) $\cot \frac{2\pi}{3} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

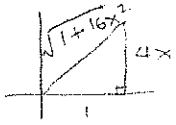
(31) $\tan\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

(32) $\sec\left(\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)\right) = \sec\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(33) $\cos\left(\sin^{-1}(2x)\right) = \sqrt{1-4x^2}$



(34) $\sec\left(\text{Arctan}(4x)\right) = \sqrt{1+16x^2}$



(35) $2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $\emptyset$

(36) $2\sin^2 x - \cos x = 1$

$2(1-\cos^2 x) - \cos x = 1$

$2 - 2\cos^2 x - \cos x = 1$

$0 = 2\cos^2 x + \cos x - 1$

$0 = (2\cos x - 1)(\cos x + 1)$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $x = \pi$

(37) $\sin(2x) = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\qquad$ $\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

(38) $2\cos(2x) + 1 = 0$

$\cos(2x) = -\frac{1}{2}$

$2x = \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n$

$x = \frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(39) $2\csc^2 x + 3\csc x - 2 = 0$

$(2\csc x - 1)(\csc x + 2) = 0$

$\csc x = \frac{1}{2}$ $\qquad$ $\csc x = -2$

$\sin x = 2$ $\qquad$ $\sin x = -\frac{1}{2}$

$\emptyset$ $\qquad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

(40) $\tan^2 x - \sec x = 1$

$\sec^2 x - 1 - \sec x = 1$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2$ $\qquad$ $\sec x = -1$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $x = \pi$

(41) $2\cos\left(\frac{x}{3}\right) - \sqrt{3} = 0$

$\cos\left(\frac{x}{3}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{3} = \frac{\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$

$x = \frac{\pi}{2} + 6\pi n, \frac{11\pi}{2} + 6\pi n$

$x = \frac{\pi}{2}$

(42) $\tan(2x) = -\sqrt{3}$

$2x = \frac{2\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

$x = \frac{\pi}{3} + \pi n, \frac{5\pi}{6} + \pi n$

$x = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$

(43) $2\sin(3x) - \sqrt{3} = 0$

$\sin(3x) = \frac{\sqrt{3}}{2}$

$3x = \frac{\pi}{3} + 2\pi n, \ \frac{2\pi}{3} + 2\pi n$

$x = \frac{\pi}{9} + \frac{2\pi n}{3}, \ \frac{2\pi}{9} + \frac{2\pi n}{3}$

$x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$

(44) $e^{2x+3} = 37$

$2x + 3 = \ln 37$

$x = \frac{-3 + \ln 37}{2}$

$x = 0.305$

(45) $e^{2x} - 5e^x + 6 = 0$

$(e^x - 2)(e^x - 3) = 0$

$e^x = 2 \qquad e^x = 3$

$x = \ln 2 \qquad x = \ln 3$

$x = 0.693 \qquad x = 1.099$

(46) $e^{2x} + 12e^{-x} - 1 = 0$ ← Multiply by e^x

$e^{2x} - e^x - 12 = 0$

$(e^x - 4)(e^x + 3) = 0$

$e^x = 4 \qquad e^x = -3$

$x = \ln 4 \qquad \emptyset$

$x = 1.386$

(47) $\frac{50}{4 + e^{2x}} = 11$

$50 = 44 + 11e^{2x}$

$e^{2x} = \frac{6}{11}$

$2x = \ln \frac{6}{11}$

$x = -0.303$

(48) $\log_4(x^2 - 3x) = 1$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x = 4 \qquad x = -1$

(49) $\ln(5x - 1) = 3$

$5x - 1 = e^3$

$x = \frac{1 + e^3}{5}$

$x = 4.217$

(50) $\log_2(x + 3) + \log_2(x - 1) = \log_2 12$

$\log_2(x^2 + 2x - 3) = \log_2 12$

$x^2 + 2x - 3 = 12$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5 \qquad x = 3$

(51) $\log_8(x + 5) - \log_8(x - 2) = 1$

$\log_8\left(\frac{x + 5}{x - 2}\right) = 1$

$\frac{x + 5}{x - 2} = 8$

$x + 5 = 8x - 16$

$21 = 7x$

$3 = x$

(52) $\log_6(\log_4(\log_2 x)) = 0$

$6^0 = 1 = \log_4(\log_2 x)$

$4 = \log_2 x$

$2^4 = 16 = x$

(53) $\log_3(\log_2(\log_5 25)) = x$

$\log_3(\log_2 2) = x$

$\log_3 1 = x$

$3^x = 1$

$x = 0$

Typo (54) $P(t) = \frac{300}{1+e^{4-t}}$

(a) $t=3 \quad P = \frac{300}{1+e^{1}} \approx$ 80.682 people

(b) $100 = \frac{300}{1+e^{4-t}}$

$100 + 100e^{4-t} = 300$

$100e^{4-t} = 200$

$e^{4-t} = 2$

$4 - t = \ln 2$

$t = 4 - \ln 2$

$t = 3.307$ days

$n = n_0 e^{kt}$

$n = 500 e^{kt}$

$1200 = 500 e^{k}$

$\frac{12}{5} = e^{k}$

$\ln \frac{12}{5} = .87546\ldots = k$

$t = 4 \quad n = 500 e^{4k}$

$n = 16,588.8$ bacteria

$8000 = 500 e^{kt}$

$16 = e^{kt}$

$\ln 16 = kt$

$t = \frac{\ln 16}{k} = 3.167$ hr

---

(56) $\lim_{x \to 3} f(x) = 4$ (57) $\lim_{x \to \infty} f(x) = 2$

(58) $\lim_{x \to 2^+} f(x) = \infty$ (59) $\lim_{x \to 0} f(x) =$ dne

(60) $\lim_{x \to -\infty} f(x) = 2$ (61) $\lim_{x \to -5^-} f(x) = 0$

---

(62) $\lim_{x \to -3} \frac{x^2+x-6}{x+3} = \lim_{x \to -3} \frac{(x+3)(x-2)}{x+3} = -5$

(63) $\lim_{x \to 0} \frac{(x-5)^2-25}{x} = \lim_{x \to 0} \frac{(x^2-10x+25)-25}{x} = \lim_{x \to 0} \frac{x(x-10)}{x} = -10$

(64) $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x \to 0} \frac{(x+1)-1}{x(\sqrt{x+1}+1)} = \frac{1}{2}$

(65) $\lim_{x \to -6} \frac{x+6}{x^2+3x-18} = \lim_{x \to -6} \frac{x+6}{(x+6)(x-3)} = -\frac{1}{9}$

(66) $\lim_{x \to -2} \frac{x^3+8}{x+2} = \lim_{x \to -2} \frac{(x+2)(x^2-2x+4)}{x+2} = 12$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

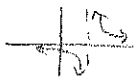
(67) $\lim_{x \to \infty} \frac{3x-5x^2}{4x^2+1} = \lim_{x \to \infty} \frac{\frac{3}{x}-5}{4+\frac{1}{x^2}} = -\frac{5}{4}$

Divide by x^2, the highest power of x in the denominator.

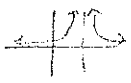
(68) $\lim_{x\to 3^+} \frac{1}{x-3} = \infty$

(69) $\lim_{x\to 3^-} \frac{1}{x-3} = -\infty$

(70) $\lim_{x\to 3} \frac{1}{x-3} =$ dne

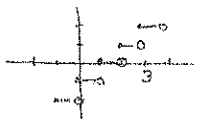


(71) $\lim_{x\to 3} \frac{1}{(x-3)^2} = \infty$

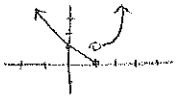


(72) $\lim_{x\to 3^+} [\![x-1]\!] = 2$

(73) $\lim_{x\to 3^-} [\![x-1]\!] = 1$

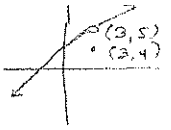


(74) $f(x) = \begin{cases} 1-x, & x \le 1 \\ x^2, & x > 1 \end{cases}$



(a) $\lim_{x\to 1^-} f(x) = 0$ (b) $\lim_{x\to 1^+} f(x) = 1$ (c) $\lim_{x\to 1} f(x) =$ dne

(75) $f(x) = \begin{cases} \frac{x^2-x-6}{x-3} = \frac{(x-3)(x+2)}{x-3} = x+2 \text{ if } x \neq 3 \\ 4 \text{ if } x = 3 \end{cases}$



(a) $\lim_{x\to 3} f(x) = 5$ (b) $f(3) = 4$

---

(80) $f(x) = 3x^4 - 5x^3 + \frac{2}{x} + 6x^{2/3} - 12 = 3x^4 - 5x^3 + 2x^{-1} + 6x^{2/3} - 12$

$f'(x) = 12x^3 - 15x^2 - 2x^{-2} + 4x^{-1/3} = 12x^3 - 15x^2 - \frac{2}{x^2} + \frac{4}{x^{1/3}}$

(81) $f(x) = \frac{2x^2-3x+1}{x} = 2x - 3 + \frac{1}{x} = 2x - 3 + x^{-1}$ ← Simplify first so you don't need to use the quotient rule.

$f'(x) = 2 - x^{-2} = 2 - \frac{1}{x^2}$

(82) $f(x) = \sqrt{x} + \sqrt[3]{x} = x^{1/2} + x^{1/3}$

$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} = \frac{1}{2x^{1/2}} + \frac{1}{3x^{2/3}}$

(83) $f(x) = (6x+5)(x^3-2)$

$f'(x) = (6x+5)(3x^2) + (x^3-2)(6) = 18x^3 + 15x^2 + 6x^3 - 12 = 24x^3 + 15x^2 - 12$

(84) $f(x) = \frac{x^3+5x-3}{x^2-1}$

$f'(x) = \frac{(x^2-1)(3x^2+5) - (x^3+5x-3)(2x)}{(x^2-1)^2} = \frac{3x^4+2x^2-5-2x^4-10x^2+6x}{(x^2-1)^2}$

$= \frac{x^4-8x^2+6x-5}{(x^2-1)^2}$

(85) $f(x) = x^4 - 3x^2 + 7$

(a) $f'(x) = 4x^3 - 6x$ (b) At $(1,5)$, $f'(1) = -2$ Tangent line: $y - 5 = -2(x-1)$

76

$2x - 8$

77

$= -\frac{3}{(x-4)^2}$

78

$= \frac{1}{2\sqrt{x+9}}$

79

$= 3x^2 + 4x - 1$