ANNUAL MATHEMATICS CONTEST SAMPLE 1 SPONSORED BY ARIZONA ASSOCIATION OF TEACHERS OF MATHEMATICS

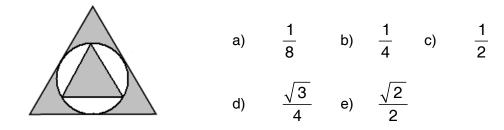
INSTRUCTIONS: Mark on the answer sheet the letter corresponding to the correct answer. Your score will be determined as follows: 4 (number of correct responses) – (number of incorrect responses). Answers left blank do not count either way. Do not guess wildly. USE A NUMBER TWO PENCIL TO MARK YOUR ANSWER SHEET. TIME OF THE CONTEST: 90 minutes. Total points 320.

1) Find an equation in slope-intercept form of the line through (-1, -4) and perpendicular to the line -8x - 5y = -12.

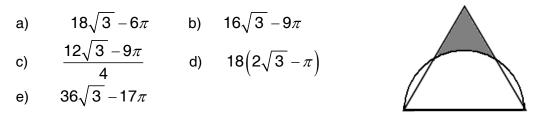
a)
$$y = \frac{1}{5}x + \frac{-12}{5}$$
 b) $y = \frac{5}{8}x + \frac{-27}{8}$ c) $y = \frac{5}{8}x + \frac{-27}{8}$
d) $y = \frac{8}{5}x + \frac{8}{5}$ e) $y = \frac{8}{5}x + \frac{-8}{5}$
2) Find the sum of the solutions to the quadratic $2x^2 - 7x + 6 = 0$
a) $-1\frac{1}{3}$ b) $3\frac{1}{2}$ c) $1\frac{1}{3}$ d) $-2\frac{2}{3}$ e) $-2\frac{1}{6}$
3) Simplify $\sqrt{6} * \sqrt{3} - 4\sqrt{50}$
a) $9 - 20\sqrt{2}$ b) $-17\sqrt{2}$ c) $6 - 20\sqrt{2}$ d) $-2\sqrt{2}$ e) $3\sqrt{3} - 20\sqrt{2}$
4) Simplify: $-10(x - 4) - 3(3 - 10x)$
a) $12x + 31$ b) $60x - 95$ c) $20x + 31$ d) $12x + 22$ e) $-40x - 49$
5) Solve for x: $10 - 4x - 7 \ge -5x - 6$
a) $x \ge -4$ b) $x \le -4$ c) $x \ge -9$ d) $x \le -9$ e) $-9 \le x \le -4$
6) Find the product and write the answer in scientific notation. $(5x10^{-6})(6.1x10^{4})$
a) 3.05×10^{-1} b) 3.05×10^{-2} c) 3.05×10^{-10} d) $1.11x 10^{-1}$ e) $1.11x 10^{-2}$
7) Evaluate the function $f(x) = \frac{2-2x}{3x^2 + 2x - 4}$, $x = -3$
a) $\frac{8}{29}$ b) $\frac{-4}{29}$ c) $\frac{-4}{17}$ d) $\frac{8}{17}$ e) 0
8) Multiply and express the product in simplest terms: $\frac{k^2 + 13k + 40}{k^2 + 10k + 25} \cdot \frac{k^2 + 7k + 10}{k^2 + 10k + 16}$
a) $\frac{k + 8}{k + 5}$ b) 1 c) $-6 < v < 5$ d) $v > 1 \text{ or } v < \frac{-1}{9}$ e) $v > -6 \text{ or } v < 5$
10) Find the value of x so that the line through the points has the given slope. $(-3, -2)and(x, -4) slope = -\frac{2}{3}$
a) 9 b) -8 c) 4 d) 0 e) 7
11) Find the sum for the solutions to the equation: $\sqrt{3x + 1} = 3 + \sqrt{x - 4}$
a) -1 b) 1 c) -13 d) 13 e) no solutions

12)	Solve for x: $-x^2 + 13x + 14 = 6x - 10$	
	a) $\left\{\frac{7\pm\sqrt{145}}{2}\right\}$ b) $\left\{\frac{-1\pm\sqrt{757}}{18}\right\}$ c) $\left\{\frac{7\pm\sqrt{73}}{2}\right\}$ d) $\left\{\frac{-7\pm\sqrt{96}}{18}\right\}$ e) no solution	
13)	A hummingbird adds 12 grams per day to its base weight of 7 grams during the spring migration. I	_et
	t(x) represents the hummingbird's weight after x days. Find $t(5.1)$ a) 19.1 grams b) 60.1 grams c) 79 grams d) 68.2 grams e) 61.2 grams	;
14)	Perform the indicated operations and simplify. $\frac{x}{x-6} + \frac{12}{x+6} - \frac{72}{x^2-36}$	
	a) $\frac{x+24}{x^2-36}$ b) 1 c) $\frac{x+24}{x+6}$ d) $\frac{x-24}{x-6}$ e) $\frac{x-24}{x^2-36}$	
15)	Solve for the variable "r": $I = \frac{E}{R+r}$	
	a) $r = \frac{E}{I} - IR$ b) $r = \frac{IR}{E}$ c) $r = \frac{E - IR}{I}$ d) $r = \frac{E - R}{I}$ e) $r = \frac{E}{IR}$	
16)	The weight of a liquid varies directly as its volume V. If the weight of the liquid in a cubical contain cm on a side is 375 g, find the weight of the liquid in a cubical container 4 cm on a side. a) 12 g b) 48 g c) 64 g d) 125 g e) 192 g	er 5
17)	If $ x-3 < 4$, then $a < x + 4 < b$. Find the values for a and b.	
	a) $a = -1$, $b = 7$ b) $a = 0$, $b = 8$ c) $a = -8$, $b = 0$ d) $a = 3$, $b = 11$ e) $a = -7$, $b = 1$	
18)	The height of a box is 7 inches. Its length is 6 inches more than its width. Find the length if the vol	ume
	is 280 cubic inches. a) 40 in. b) 7 in. c) 10 in. d) 4 in. e) 13 in.	
19)	What is the approximate diameter of a wheel that made 752 revolutions when it rolled 1 kilometer a) 24.2 cm b) 32.0 cm c) 42.3 cm d) 84.6 cm e) 113 cm	?
20)	Approximately how much larger is a 14 inch (circular) pizza than a 12 inch (circular) pizza? a) 16.7% b) 19.7% c) 25% d) 30.6% e) 36.1%	
21)	If the ratios between the adjacent angles of a pentagon are 3:4:5:6:7, what is the size of the small	est
	angle? a) 14.4° b) 21.6° c) 64.8° d) 90° e) 108°	
22)	What is the volume of the largest cube that can fit inside a sphere with a radius of 6 units?	
	a) $96\sqrt[3]{39\pi}$ b) $12\sqrt[3]{364\pi}$ c) 288π d) 216 e) $192\sqrt{3}$	
23)	Find the area of a right triangle which has a perimeter of length 16 meters and a hypotenuse with length of 7 meters.	a
c (1)	a) $\sqrt{56} m^2$ b) $8 m^2$ c) $10 m^2$ d) $12 m^2$ e) $16 m^2$	
24)	Two boxes each have a volume of 1728 cubic inches. Box A is a cube, and box B has dimensions 12 x 36. By what factor is the surface area of box B larger than box A?	, 4 x
	a) 1.2 b) 1.25 c) $1.\overline{3}$ d) 1.4 e) $1.\overline{4}$	

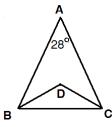
25) Consider a circle inscribed in an equilateral triangle, which circumscribes a smaller equilateral triangle. What proportion is the smaller triangle's area to the larger one's?



26) Consider an equilateral triangle that shares its base with a semicircle that has a diameter of 12 units. What is the area of the region in the triangle not covered by the semicircle?



- A latex balloon with a spherical shape has a diameter of six inches. Approximately how much air must 27) be blown into the balloon so that it doubles its surface area?
 - d) 226*in*³ c) 791*in*³ e) 320*in*³ b) 452*in*³ 207 in³ a)
- An underground oil tank is a right circular cylinder of radius 6 feet and a length of 12 feet. The tank is 28) buried horizontally with the axis of the cylinder parallel to the surface of the ground. If the depth of the oil in the tank is 3 feet (at its deepest point), what is the volume of the oil in the tank to the nearest cubic foot?
 - a) 261 c) 263 d) 265 b) 262 264 e)
- In the figure AB = AC, BD bisects angle ABC and CD bisects angle ACB. Find the number of degrees 29) in angle BDC.
 - 100° b) 104° c) 126° 136° e) 56° a)
 - d)

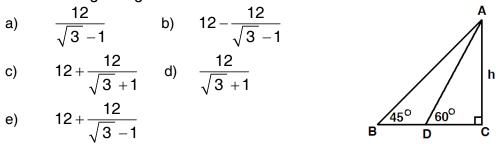


30) The diagonal of a square is a+b. The side of the square is...

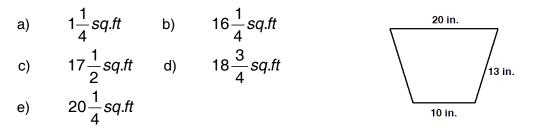
a)
$$\frac{a+b}{2}$$
 b) $\sqrt{a+b}$ c) $\frac{a+b}{\sqrt{2}}$ d) $\frac{(a+b)^2}{2}$ e) $\frac{\sqrt{a+b}}{2}$

- 31) The circumference of the circular pool is 8 meters less than the outer circumference of a walk surrounding the pool. What is the width of the walk?
 - a) $\frac{4}{\pi}m$ b) $\frac{\pi}{4}m$ c) $\frac{1}{2}\pi m$ d) $\frac{8}{\pi}m$ e) $\frac{\pi}{8}m$
- The distance from home plate to dead center field in a certain baseball stadium is 402 feet. A 32) baseball diamond is a square with a distance from home plate to first base of 90 feet. How far is it from first base to dead center field?
 - b) 470 ft 274.7 ft c) 344.3 ft d) 327.2 ft e) 379.6 ft a)

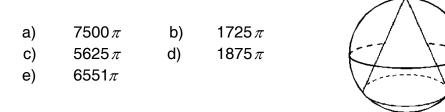
33) In the following triangles find AC if BD = 12



34) The ends of a wooden trough are isosceles trapezoids as shown. The trough length is 5 feet long. How much wood is used in constructing the trough? The trough is open on the top.



- 35) A parallelogram has a base of length x + 7 and a height of x + 2 and has an area of 50 square units. Find the base of the parallelogram.
 - a) 8 units b) 10 units c) 15 units d) 25 units e) 50 units
- 36) A cone is inscribed in a sphere. The sphere has a radius of 17. The radius of the cone's base is 15. The height of the cone exceeds 17. Find the volume of the cone



37) A water tank can be filled in 8 minutes and emptied in 9 minutes. If the drain is accidentally left open when the tank is being filled, how long does it take to fill the tank?

a) $\frac{1}{72}$ min b) $\frac{1}{17}$ min c) $4\frac{4}{17}$ min d) 14 min e) 72 min

- Find the area of the triangle formed whose vertices have the coordinates of (2,1), (6, 3), and (4, 7).
 a) 21.5 sq units b) 10 sq units c) 14 sq units d) 8 sq units e) 17 sq units
- 39) A man rode a bicycle for 12 miles and then hiked an additional 8 miles. The total time for the trip was 5 hours. If his rate when riding his bicycle was 10 mph faster than his rate walking, what was each rate?

a)	Bike: 13 mph Hike: 3 mph	b)	Bike: 11.5 mph Hike 1.5 mph	c)	Bike: Hike:	•
d)	Bike: 14.5 mph Hike: 4.5 mph	e)	Bike: 15 mph Hike 5 mph			

- 40) Two cars leave an intersection. One car travels north; the other east. When the car traveling north had gone 15 miles, the distance between the cars was 5 miles more than the distance traveled by the car heading east. How far had the eastbound car traveled?
 - a) 20 miles b) 30 miles c) 15 miles d) 25 miles e) 35 miles

- A new homeowner has a triangular-shaped back yard. Two of the three sides measure 65 ft and 80 ft 41) and form an included angle of 125°. The owner wants to approximate the area of the yard, so that he can determine the amount of fertilizer and grass seed to be purchased. Find the area of the yard rounded to the nearest square foot.
 - 4260 sq ft b) 2140 sq ft c) 5200 sq ft 2130 sq ft a) d) e) 3645 sq ft
- Find the measure of the largest angle in the triangle with sides 5, 6, and 9. 42)
 - 38.9° 81.6° d) 101.6° 31.6° C) 109.6° a) b) e)

A ship sailing parallel to shore sights a lighthouse at an angle of 10° from its direction of travel. After 43) traveling 5 miles farther, the angle is 24°. At that time, how far is the ship from the lighthouse?

- b) 3.59 mi c) 5 mi d) 6.2 mi 8.41 mi a) 2.13 mi e)
- The amount of time it takes a swimmer to swim a race is inversely proportional to the average speed 44) of the swimmer. A swimmer finishes a race in 50 seconds with an average speed of 3 feet per second. Find the average speed of the swimmer if it takes 30 seconds to finish the race.
- a) 4 fps b) 5 fps C) 6 fps d) 7 fps e) 8 fps Simplify: $\sqrt{\frac{2^{x+4}-2(2^{x+1})}{2(2^{x+3})}}$ 45) $\frac{3}{8}$ b) $\frac{\sqrt{3}}{4}$ d) $\frac{x\sqrt{3}}{4}$ e) $\frac{\sqrt{3}}{2}$ c) 2^x a)

46) Solve this equation for a:
$$\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$$

a) $b + c$ b) $\frac{bc}{b+c}$ c) $\frac{b+c}{bc}$ d) $\frac{bc}{b-c}$ e) $\frac{b-c}{bc}$

- Find the function that is finally graphed after the following transformations are applied to the graph of 47) y = |x|. The graph is shifted right 3 units, stretched by a factor of 3, shifted vertically down 2 units, and then finally reflected across the x-axis.
 - a) y = -(3|x-3|-2) b) y = -3|x-3|-2 c) y = -(3|x+3|-2)y = 3|-x-3|-2 e) y = -3|x+3|+2d)

Find the equation of the oblique asymptote, if any, for the function $f(x) = \frac{x^2 - 9x + 5}{x + 3}$ 48) y = x + 14 b) y = x - 12 c) y = x - 9 d) y = x - 6 e) no oblique a) Find the sum of the roots of: $e^{2x} - 7e^x + 10 = 0$ a) ln(10) b) ln(7) c) 7 49)

How long will it take for an investment to triple in value if it earns 8.25% compounded continuously? 50)

ln(10)

a)

b) ln(7)

13.317 years b) 6.658 years c) 8.402 years d) 14.244 years e) 10.235 years a) Solve the equation for x: $e^{x+2} = 8$ 51)

d)

5ln(2)

e)

2ln(5)

b) e¹⁶ $e^{8} + 2$ e) $e^8 - 2$ C) ln(8) – 2 a) ln(10) d)

Solve the equation: $\log_{30}(x^2 - x) = 1$ 52) $\{5, 6\}$ b) $\{1, 30\}$ c) $\{-5, -6\}$ d) $\{-30, -1\}$ e) {-5, 6} a)

- A reflecting telescope contains a mirror shaped like a paraboloid of revolution. If the mirror is 20 53) inches across at its opening and is 5 feet deep, where, approximately, will the light be concentrated?
 - 5 inches from the vertex a)
- b) 0.6 inches from the vertex
- c) 0.4 inches from the vertex d)
- 1.3 inches from the vertex
- 1.6 inches from the vertex e)
- 54) A bridge is built in the shape of a semielliptical arch. It has a span of 104 feet. The height of the arch 30 feet from the center is to be 10 feet. Find the height of the arch at its center. a)
 - 30.57 feet 12.24 feet 10.44 feet 17.33 feet 15.25 feet b) C) d) e)
- 55) The odds in favor of a horse winning a race are posted as 9:4. Find the probability that the horse will win the race.
 - 9 13 b) $\frac{4}{9}$ c) $\frac{4}{13}$ d) $\frac{9}{14}$ e) $\frac{9}{4}$ a)

A ten question multiple choice test has four possible answers for each question. A student randomly 56) guesses, what the probability of getting at least six correct answers? .989 b) .995 C) .118 .020 e) .250 a) d)

57) What is the range for the following piecewise function?

$$f(x) = \begin{cases} 4, & if - 5 \le x < -3 \\ |x|, & if - 3 \le x < 6 \\ \sqrt{x}, & if 6 \le x \le 14 \end{cases}$$
 a) $\begin{bmatrix} 0, 6 \end{bmatrix}$ b) $\begin{bmatrix} 0, \sqrt{14} \end{bmatrix}$ c) $\begin{bmatrix} 0, \infty \end{bmatrix}$

- A class has 10 boys and 12 girls. In how many ways can a committee of four be selected if the 58) committee can have at most two girls?
 - a) 4620 ways b) 5170 ways c) 4410 ways d) 5665 ways e) 7315 ways
- 59) A bag contains 6 cherry, 3 orange, and 2 lemon candies. You reach in and take 3 pieces of candy at random. Find the probability of drawing 1 cherry and 2 lemons.
 - a) 1 b) .0364 c) .3636 d) .0424 e) .0303
- If the tide changes from high tide to low tide in a six hour period, and the average depth of a certain 60) river is 14 m and ranges from 11 to 17 m, approximate the depth of the river by a sine curve. Which is the equation that gives the depth x hours after midnight given that high tide occurs at 6:00am.

a)
$$d = 3\sin\left(\frac{\pi x}{6} - \frac{\pi}{2}\right) + 14$$
 b) $d = 3\sin\left(\frac{\pi x}{12}\right) + 14$ c) $d = 3\sin\left(\frac{\pi x}{6} - 6\right) + 14$
d) $d = 6\sin\left(\frac{\pi x}{6} - \frac{\pi}{4}\right)$ e) $d = 6\sin\left(\frac{\pi x}{12} - \frac{\pi}{4}\right)$

- If an object is propelled upward from a height of 112 feet at an initial velocity of 96 feet per second, 61) then its height h after t seconds is given by the equation $h = -16t^2 + 96t + 112$ After how many seconds does the object hit the ground?
 - 3.5 b) 7 d) 10 a) 6 C) e) 11

Evaluate: $\left[2\left(\cos 72^\circ + i\sin 72^\circ\right)\right]^5$ 62) a) 10 b) C) 160 + 5*i* d) 32 + i e) 10 + 5i $r = \frac{9}{3 - 3\cos(\theta)}$ 63) Convert the polar equation to a rectangular equation. b) $y^2 = -6x + 9$ c) $x^2 = 6v + 9$ a) $x^2 = -6y + 9$ $y^2 = 6x + 9$ e) $x^2 = -6v - 9$ d) Which polynomial with real coefficients can be described as having a degree of three and zeros of 64) -2 and 3 + i? $f(x) = x^3 - 4x^2 - 10x + 20$ b) $f(x) = x^3 - 4x^2 - 2x + 20$ c) $f(x) = x^3 - 8x^2 + 2x + 20$ a) $f(x) = x^3 - 8x^2 - 10x + 20$ e) $f(x) = x^3 - 6x^2 - 2x + 20$ d) Solve Let P = (x, y) be a point on the graph of $y = \sqrt{x}$. Express the distance d from P to the point 65) (1,0) as a function of x. $d(x) = x^2 - x + 1$ $d(x) = \sqrt{x^2 - x + 1}$ b) $d(x) = \sqrt{x^2 - x + 1}$ c) $d(x) = x^2 + 2x + 2$ $d(x) = \sqrt{x^2 + 2x + 2}$ e) $d(x) = \sqrt{x^2 + 1}$ a) $d(x) = x^2 - x + 1$ d) A radio transmission tower is 170 feet tall. How long should a guy wire be if it is to be attached 8 feet 66) from the top and is to make an angle of 29 degrees with the ground? b) 194.4 ft 334.2 ft 254.3 ft a) 350.7 ft C) 185.2 ft d) e) 67) Find the length of side c to the nearest hundredth for the triangle with the given angles and AC = 8. С 4.77 b) 6.13 c) 15.04 a) B 20° 110° 8 17.92 d) e) 21.98 $\frac{1+\cot^2(x)}{\sin(2x)}$ Simplify: 68) a) $2\sin^{3}(x)\cos(x)$ b) $\frac{1}{2}\sec^{3}(x)\csc(x)$ c) $2\csc^3(x)\cos(x)$ d) $\frac{1}{2} \csc^{3}(x) \sec(x)$ e) $\frac{1}{2} \cot^{3}(x) \csc(x)$ If the $sin(A) = \frac{4}{5} and cos(B) = \frac{12}{13}$. A and B are acute angles. Find sin(A - B)69) b) $\frac{33}{65}$ c) $\frac{56}{65}$ d) $\frac{16}{65}$ e) $\frac{-16}{65}$ a) The partial fraction decomposition of the rational expression $\frac{4x-13}{2x^2+x-6} = \frac{A}{x+2} + \frac{B}{2x-3}$ find the 70) sum of A and B. b) – 1 c) 0 d) 1 a) -9 e) 9 Find the slope of the line tangent to the curve at the given point. $y = 11\sin(x); x = \frac{\pi}{3}$ 71) d) $\frac{11\sqrt{3}}{2}$ e) $\frac{\sqrt{3}}{2}$ b) $\frac{-11}{2}$ c) $\frac{11}{2}$ a)

- 72) Find the cross product $v \times w$, when v = 2i 3j, w = -5i 4j 2k. a) -8i + 10j - 26k b) -23i - 6j - 4k c) 6i - 4j - 20k
 - d) 6i + 4j + 7k e) 6i + 4j 23k

73) Find the second derivative of the function
$$f(x) = \sqrt{3x-7}$$

a) $\frac{10}{4(3x-7)^{\frac{3}{2}}}$ b) $\frac{9}{4(3x-7)^{\frac{3}{2}}}$ c) $\frac{-10}{4(3x-7)^{\frac{3}{2}}}$ d) $\frac{-9}{4(3x-7)^{\frac{3}{2}}}$ e) $\frac{10}{4(3x-7)^{\frac{5}{2}}}$
74) Find the integral: $\int -6x^4 \sin(x^5) dx$

- a) $\frac{6}{5}x^5\cos(x^5)+c$ b) $6\cos(x^5)+c$ c) $\frac{6}{5}\cos(x^5)+c$ d) $30\cos(x^5)+c$ e) $30x^5\cos(x^5)+c$
- 75) Find the volume of the solid of revolution formed by rotating about the x-axis the region bounded by the curves: $f(x) = \sqrt{x}$, y = 0, x = 1, and x = 18a) $8.5\pi u^3$ b) $323\pi u^3$ c) $200\pi u^3$ d) $161.5\pi u^3$ e) $172\pi u^3$
- 76) What is the *y*-intercept of the line that is tangent to the curve $y = e^{kx}$ at the point where $x = \frac{1}{2k}$? a) $\frac{e^{\frac{1}{2}}}{2}$ b) $e^{\frac{1}{2}}$ c) $\frac{k}{2}$ d) k e) $ke^{\frac{1}{2}}$

77) A water tank that contains 10 gallons initially develops a leak. The rate at which water is leaking from the tank is modeled by the function: $R(t) = 2 - \frac{t}{(t^2 + 9)^{\frac{1}{2}}}$, where *R* is measured in gallons per hour and *t* is measured in hours, starting from the time the tank began to leak. How much water remains in the tank after 4 hours?

- a) 0 b) 2 c) 4 d) 6 e) 8
- 78) The equation of the tangent line to the curve $y = x^3 3x^2 + 4$ at its point of inflection is
 - a) y = -3x 1 b) y = 3x 1 c) y = -3x + 5 d) y = 3x + 5 e) y = -3x 18

79) If
$$F(x) = \int_{0}^{X} \sqrt{t^{3} + 1} dt$$
, then $F'(2) =$
a) -3 b) -2 c) 2 d) 3 e) 18

- 80) Let $f(x) = x^2 \cdot \sin(\frac{\pi}{4}x)$ for all values of x. Suppose the tangent line to f at the point where x = 2 is used to estimate f(2.01). What is the value of this estimate?
 - a) 0 b) .01 c) $\frac{\pi}{2}$ + 2.01 d) 4.04 e) 8.04

Here are the answers.

4	<u> </u>	•	D	•	D	4	0
1.	С	2.	В	3.	В	4.	С
5.	С	6.	Α	7.	D	8.	В
9.	D	10.	D	11.	D	12.	Α
13.	D	14.	С	15.	С	16.	Е
17.	D	18.	С	19.	С	20.	Е
21.	С	22.	Е	23.	В	24.	Е
25.	В	26.	Α	27.	Α	28.	Е
29.	В	30.	С	31.	А	32.	С
33.	Е	34.	С	35.	В	36.	D
37.	Е	38.	В	39.	С	40.	Α
41.	D	42.	Е	43.	В	44.	В
45.	Е	46.	В	47.	А	48.	В
49.	А	50.	А	51.	С	52.	Е
53.	С	54.	В	55.	А	56.	D
57.	А	58.	А	59.	В	60.	Α
61.	С	62.	В	63.	D	64.	В
65.	В	66.	D	67.	D	68.	D
69.	В	70.	D	71.	С	72.	Е
73.	D	74.	С	75.	D	76.	Α
77.	С	78.	С	79.	D	80.	D