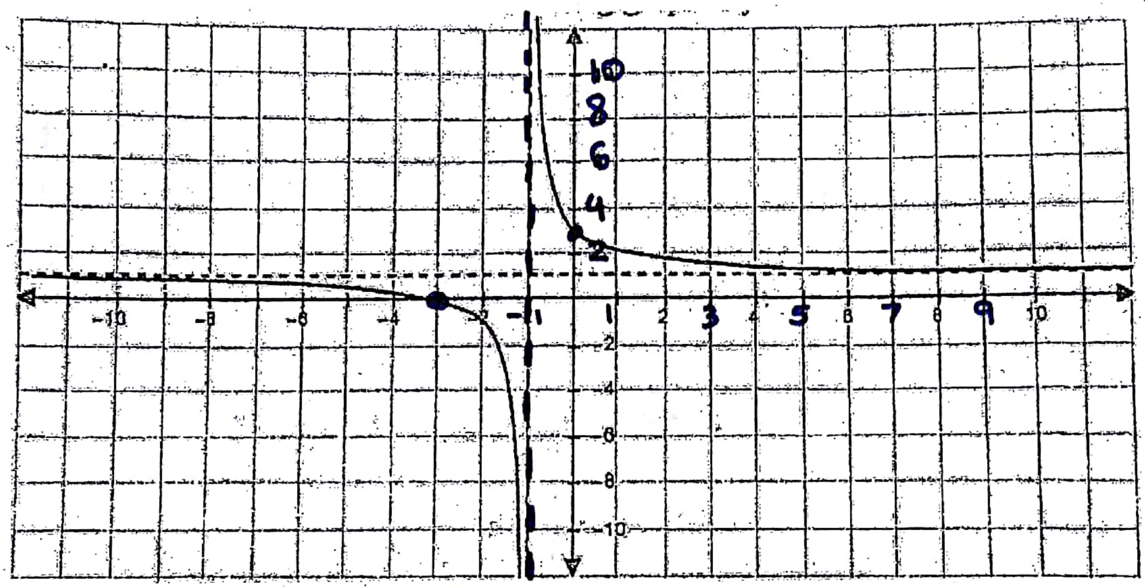
Rational Functions

y.int x.int V.A. H.A.

Given the following graph, identify the vertical and horizontal intercepts, and the vertical and horizontal asymptotes of the function. Then, determine a rule for the function.



V. A. $x = -1$ (denominator)

check ← H.A $y = 1$ (N & D have same degree)

x. int $(-3, 0)$ $(N = 0)$

y.int $(0, 3)$

use y.int to figure out a

$f(0) = 3$

$$f(x) = a\left(\frac{x+3}{x+1}\right)$$

$$3 = a \cdot \left(\frac{0+3}{0+1}\right)$$

$$\frac{3}{3} = \frac{3a}{3}$$

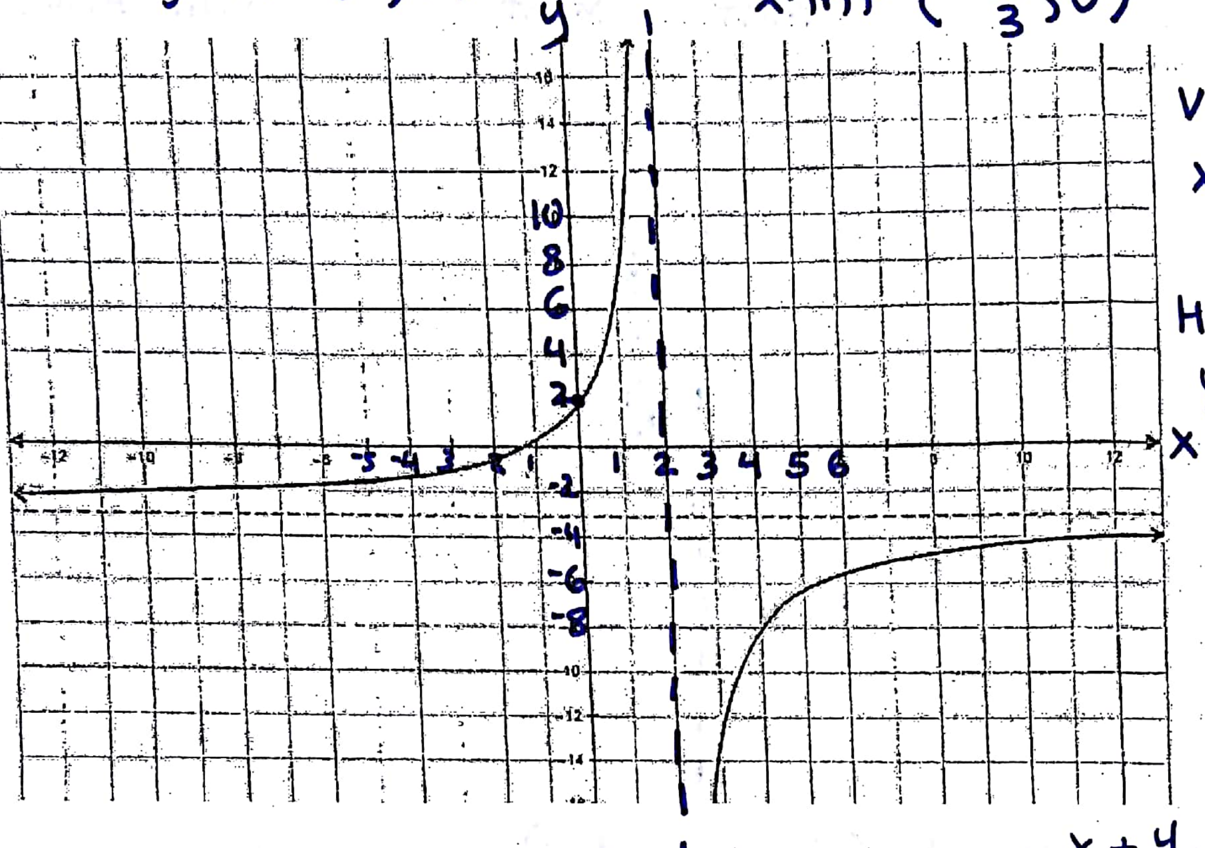
$$1 = a$$

$$\boxed{f(x) = \frac{x+3}{x+1}}$$

$f(x) =$

Given the following graph, identify the vertical and horizontal intercepts, and the vertical and horizontal asymptotes of the function. Then, determine a rule for the function.

y·int $(0, 2)$ x·int $\left(-\frac{4}{3}, 0\right)$



V.A. $x = 2$

H.A. $y = -3$

$$f(x) = a\left(\frac{3x+4}{x-2}\right)$$

$$x + \frac{4}{3}$$

$$3x + 4 = 0$$
$$-4 \quad -4$$
$$\frac{3x}{3} = \frac{-4}{3}$$

$$2 = a\left(\frac{3 \cdot 0 + 4}{0 - 2}\right)$$

$$\frac{2}{-2} = \frac{-2a}{-2}$$

$$-1 = a$$

$$f(x) = \frac{-1}{1}\left(\frac{3x+4}{x-2}\right)$$

$$\boxed{f(x) = \frac{-3x-4}{x-2}}$$

We also know there is a vertical asymptote at $x=-2$, and by plugging in a few values or looking at a graph we can determine that $f(x)\to-\infty$ as $x\to-2^-$. Therefore, we can condense $f(x)\to-\infty$ as $x\to-2^-$ using limit notation by writing $\lim_{x\to-2^-} f(x)=-\infty$. Similarly, $f(x)\to\infty$ as $x\to-2^+$ so $\lim_{x\to-2^+} f(x)=\infty$.

Use the following functions to answer the questions below. (Note: If a value does not exist, write DNE.)

$f(x)=\dfrac{x-3}{x+2}$ $\quad g(x)=\dfrac{3x^2}{(x-1)(x-3)}$ $\quad h(x)=\dfrac{x^2+1}{x-2}$ $\quad k(x)=\dfrac{5x}{x^2-4}$

a. $\lim_{x\to\infty} f(x)=$ 1
$\lim_{x\to-\infty} f(x)=$ 1
$\lim_{x\to-2^-} f(x)=$ ∞
$\lim_{x\to-2^+} f(x)=$ $-\infty$

b. $\lim_{x\to\infty} g(x)=$ 3
$\lim_{x\to1^-} g(x)=$ ∞
$\lim_{x\to1^+} g(x)=$ $-\infty$
$\lim_{x\to3^+} g(x)=$ ∞

c. $\lim_{x\to\infty} h(x)=$ ∞
$\lim_{x\to-\infty} h(x)=$ $-\infty$
$\lim_{x\to2^-} h(x)=$ $-\infty$
$\lim_{x\to2^+} h(x)=$ ∞

d. $\lim_{x\to-\infty} k(x)=$ 0
$\lim_{x\to-2^-} k(x)=$ $-\infty$
$\lim_{x\to2^-} k(x)=$ $-\infty$
$\lim_{x\to2^+} k(x)=$ ∞

7. In each part below, you are given information about a rational function. Use the information to sketch a possible graph of the function.

a. $\lim_{x\to\infty} f(x)=-3$
$\lim_{x\to-\infty} f(x)=-3$
$\lim_{x\to2^-} f(x)=\infty$
$\lim_{x\to2^+} f(x)=-\infty$

b. ~~$\lim_{x\to\infty} f(x)$ DNE~~
~~$\lim_{x\to\infty} f(x)$ DNE~~
~~$\lim_{x\to-4^-} f(x)=-\infty$~~
~~$\lim_{x\to-4^+} f(x)=\infty$~~

c. ~~$\lim_{x\to\infty} f(x)=2$~~
~~$\lim_{x\to-\infty} f(x)=2$~~
~~$\lim_{x\to-3^-} f(x)=\infty$~~
~~$\lim_{x\to-3^+} f(x)=-\infty$~~
~~$\lim_{x\to1^-} f(x)=-\infty$~~
~~$\lim_{x\to1^+} f(x)=\infty$~~

a.

H.A.
$y=-3$

V.A
$x=2$