5.3 Notes: Sum and Difference Formulas

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

<u>**Investigation:**</u>

1. A) Do you think the following expressions equivalent?

$$\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) \neq \cos\frac{\pi}{4} + \cos\frac{\pi}{3}$$

cannot distribute a function

$\cos(u+v)$

B) Find the exact value of $\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$ using a sum identity from above.

$$\cos\frac{\pi}{4}\cdot\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\cdot\sin\frac{\pi}{3}$$

$$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

C) Find the exact value of $\cos\frac{\pi}{4} + \cos\frac{\pi}{3}$

$$\frac{\sqrt{2}}{2} + \frac{1}{2} = \boxed{\frac{\sqrt{2}+1}{2}}$$

$\neq$

D) Are the two expressions above equivalent? No!

2. Break up the following angle into the sum or difference of two angles on the unit circle

$$\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12} = \left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$\frac{8\pi}{12} - \frac{3\pi}{12} = \left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$$

Ex: 1 Find the exact value of: a.) $\sin\frac{5\pi}{12}$

a) $\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{6}+\cos\frac{\pi}{4}\cdot\sin\frac{\pi}{6}$

$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

b.) $\tan\frac{5\pi}{12}=\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=$

$\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{\pi}{6}}$

$\frac{\left(1+\frac{\sqrt{3}}{3}\right)\cdot\left(1+\frac{\sqrt{3}}{3}\right)}{\left(1-1\cdot\frac{\sqrt{3}}{3}\right)\left(1+\frac{\sqrt{3}}{3}\right)}$

$\frac{1+\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{3}+\frac{1}{3}}{1+\cancel{\frac{\sqrt{3}}{3}}-\cancel{\frac{\sqrt{3}}{3}}-\frac{1}{3}}$ $\quad -\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{3}$

$\frac{\frac{4}{3}+\frac{2\sqrt{3}}{3}}{2/3}=\frac{\frac{(4+2\sqrt{3})}{\cancel{3}}\cdot\frac{\cancel{3}}{2}}{\frac{2}{3}}$

$\frac{4+2\sqrt{3}}{2}$

$\boxed{2+\sqrt{3}}$

You try: find the exact values of $\cos\frac{5\pi}{12}$

$\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$\cos\frac{\pi}{4}\cdot\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\cdot\sin\frac{\pi}{6}$

$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$ $\quad\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Ex: 2 Find the exact value of a.) sin 195° b.) cos 195°

$195°=150°+45°$

$=60°+135°$

$=225°-30°$

$=240°-45°$

$=315°-120°$

a) $\sin(60+135)=$

$\sin 60\cdot\cos 135+\cos 60\cdot\sin 135$

$\frac{\sqrt{3}}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)+\frac{1}{2}\cdot\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

b) $\cos(60+135)=$

$\cos 60\cdot\cos 135-\sin 60\cdot\sin 135$

$\frac{1}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

You try: find the exact value of tan 195°

$\tan(60+135)=$

$\frac{\tan 60°+\tan 135}{1-\tan 60\cdot\tan 135}=\frac{\sqrt{3}+-1}{1-(\sqrt{3})(-1)}$

$\frac{(\sqrt{3}-1)}{(1+\sqrt{3})}\cdot\frac{(1-\sqrt{3})}{(1-\sqrt{3})}=\frac{\sqrt{3}-3-1+\sqrt{3}}{1-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-3}=\frac{-4+2\sqrt{3}}{-2}$

$\frac{-4}{-2}+\frac{2\sqrt{3}}{-2}$

$\boxed{2-\sqrt{3}}$

5.3 Notes: Sum and Difference Formulas

Ex: 3 Find the exact value of: a.) $\sin\frac{\pi}{12}$ b.) $\tan\frac{\pi}{12}$

$\frac{3\pi}{12} - \frac{2\pi}{12}$

a) $\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

b) $\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

a) $\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b) $\boxed{2-\sqrt{3}}$

You try: Find the exact value of $\cos\frac{\pi}{12}$.

$\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Ex: 4 Find the exact value of: a.) $\cos 75°$ b.) $\tan 75°$

a) $\cos(45° + 30°)$

b) $\tan(45° + 30°)$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\boxed{2+\sqrt{3}}$

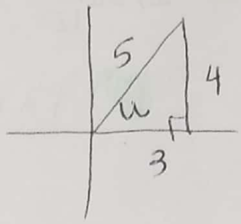
You try: Find the exact value of $\sin 75°$.

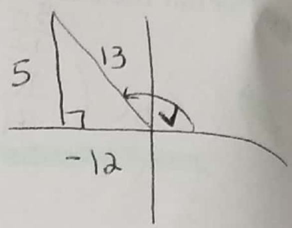
$\sin(45° + 30°)$

$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Calc ok

Ex: 5 **Find the exact value of** $\sin(u+v)$ **given** $\sin u = \frac{4}{5}$, **where** $0 < u < \frac{\pi}{2}$, **and** $\cos v = -\frac{12}{13}$, **where** $\frac{\pi}{2} < v < \pi$.



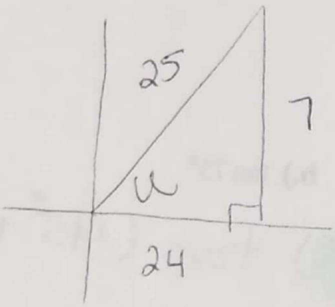


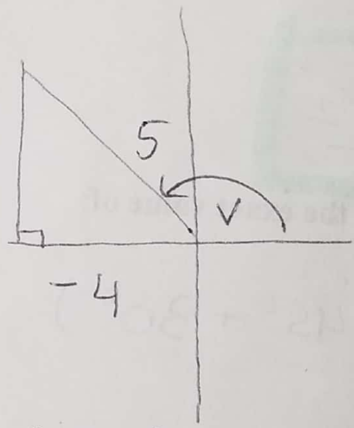
$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\frac{4}{5} \cdot \frac{-12}{13} + \frac{3}{5} \cdot \frac{5}{13}$$

$$\frac{-48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$$

You Try: **Find the exact value of** $\cos(u+v)$ **given** $\sin u = \frac{7}{25}$, **where** $0 < u < \frac{\pi}{2}$, **and** $\cos v = -\frac{4}{5}$, **where** $\frac{\pi}{2} < v < \pi$.





$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\frac{24}{25} \cdot \frac{-4}{5} - \frac{7}{25} \cdot \frac{3}{5}$$

$$\frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}}$$

Ex: ~~4~~ 6 a.) Use a difference formula to prove the cofunction identity $\cos\left(\frac{\pi}{2}-x\right)=\sin x$.

$\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$0\cdot\cos x + 1\cdot\sin x$

$\sin x = \sin x$ ✓

b.) Prove the identity $\tan(x-\pi)=\tan x$

$$\frac{\tan x - \tan\pi}{1+\tan x\cdot\tan\pi}$$

$$\frac{\tan x - 0}{1+\tan x\cdot 0}$$

$\tan x = \tan x$

You Try: Use a difference formula to prove the cofunction identity $\sin\left(x-\frac{\pi}{2}\right)=-\cos x$.

$\sin x\cos\frac{\pi}{2} - \cos x\sin\frac{\pi}{2}$

$\sin x\cdot 0 - \cos x\cdot 1$

$-\cos x = -\cos x$ ✓

Ex: 7 Simplify each expression a.) $\cos\left(\theta-\frac{3\pi}{2}\right)$

$\cos\theta\cos\frac{3\pi}{2} + \sin\theta\sin\frac{3\pi}{2}$

$\cos\theta\cdot 0 + \sin\theta\cdot(-1)$

$\boxed{-\sin\theta}$

b.) $\tan(\theta+3\pi)$

$$\frac{\tan\theta + \tan 3\pi}{1-\tan\theta\cdot\tan 3\pi}$$

$$\frac{\tan\theta + 0}{1-\tan\theta\cdot 0} = \boxed{\tan\theta}$$

You Try: Simplify the expression $\sin(5\pi-\theta)$.

$\sin 5\pi\cos\theta - \cos 5\pi\sin\theta$

$0\cdot\cos\theta - (-1)\cdot\sin\theta$

$0 - (-\sin\theta) = \boxed{\sin\theta}$