Area of an Oblique Triangle

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The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B.$$

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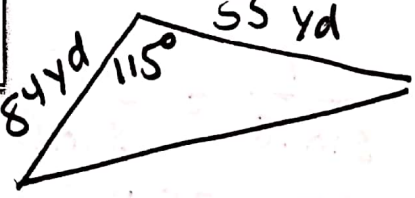


Example 6: Finding the Area of a Triangular Lot

Find the area of a triangular lot containing side lengths that measure 84 yards and 55 yards and form an angle of 115°.

$A = \frac{1}{2}(84)(55)\sin 115°$

2093.6 yd^2

You try:

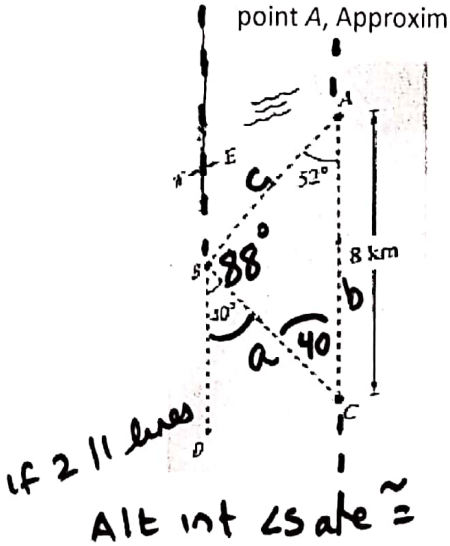
Find the area of a triangular lot having two sides of lengths 24 inches and 18 inches and an included angle of 80°.

$A = \frac{1}{2}(24)(18)\sin 80$

212.7 in^2

Example 7: An Application of the Law of Sines

a. The course for a boat races starts at point *A* and proceeds in the direction S 52° W to point *B*, then in the direction S 40° E to point *C*, and finally back to point *A*. Point *C* lies 8 kilometers directly south of point *A*, Approximate the total distance of the race course. See figure below



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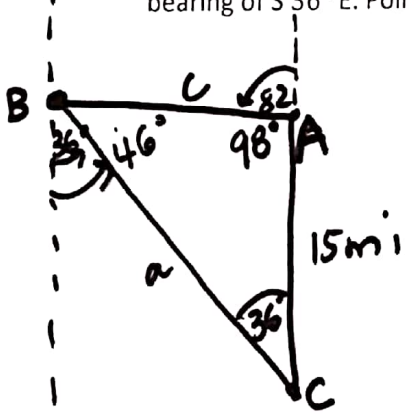
$\frac{8}{\sin 88} = \frac{a}{\sin 52}$ $\quad a = 6.31$ km

$\frac{8}{\sin 88} = \frac{c}{\sin 40}$ $\quad c = 5.15$ km

total dist $a+b+c$ = 19.45 km

b. A boat travels from point *A* to point *B* at a bearing of N 82° W. The boat then travels to point *C* at a bearing of S 36° E. Point *C* is 15 miles due south of point *A*. How many total miles does the boat travel?

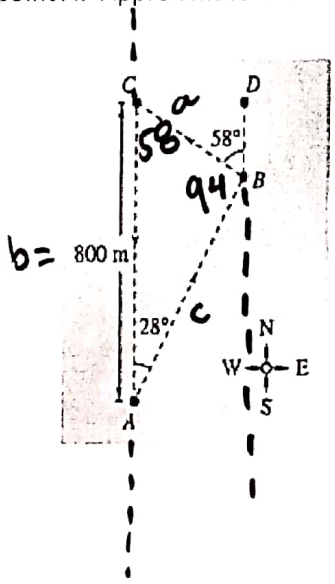


$\frac{15}{\sin 46} = \frac{a}{\sin 98}$ $\quad a = 20.65$ mi

$\frac{15}{\sin 46} = \frac{c}{\sin 36}$ $\quad c = 12.26$ mi

$a + c = 32.91$ mi

You try: On a small lake, you swim from point *A* to point *B* at a bearing of N 28° E, then to point *C* at a bearing of N 58° W, and finally back to point *A*, as shown in the figure below. Point *C* lies 800 meters directly north of point *A*. Approximate the total distance that you swim.



$\frac{800}{\sin 94} = \frac{a}{\sin 28}$ $\quad a = 376.49$ m

$\frac{800}{\sin 94} = \frac{c}{\sin 58}$ $\quad c = 680.10$ m

$a + b + c = 1856.59$ m